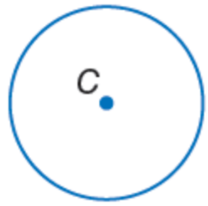
Geo C9 handouts to put in notes

Section 9.1



Circle C or $\odot C$

KEY TERMS

Circle -The locus or set of all points in a plane equidistant from a given point called the center.

Coplanar - Points that lie in the same plane.

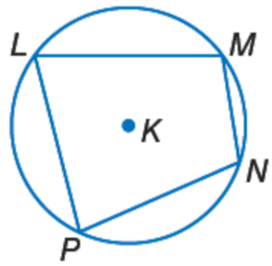
Circumference - The distance around a circle.

pi (π) - An irrational number that is the ratio of $\frac{\text{Circumference}}{\text{Diameter}}$.

Inscribed - A polygon is inscribed in a circle if all of its vertices lie on the circle.

Circumscribed - A circle is circumscribed about a polygon if it contains all of the vertices of the polygon.

- Quadrilateral LMNP is inscribed in circle K.
- Circle K is circumscribed about quadrilateral LMNP.



StudyTip

Levels of Accuracy Since π is irrational, its value cannot be given as a terminating decimal. Using a value of 3 for π provides a quick estimate in calculations. Using a value of 3.14 or $\frac{22}{7}$ provides a closer approximation. For the most accurate approximation, use the π key on a calculator. Unless stated otherwise, assume that in this text, a calculator with a π key was used to generate answers.

StudyTip

Circumcircle A *circumcircle* is a circle that passes through all of the vertices of a polygon.

By definition, the distance from the center of a circle to any point on the circle is always the same. Therefore, all radii r of a circle are congruent. Since a diameter d is composed of two radii, all diameters of a circle are also congruent.

The segment connecting the centers of the two intersecting circles contains the radii of the two circles.

Geo C9 handouts to put in notes

KeyConcept Special Segments in a Circle

Section 9.1

A **radius** (plural radii) is a segment with endpoints at the center and on the circle.

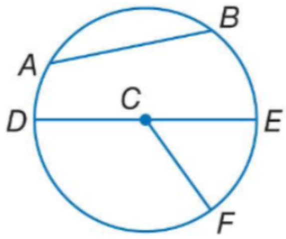
Examples $\overline{CD}$, $\overline{CE}$, and $\overline{CF}$ are radii of $\odot C$.

A **chord** is a segment with endpoints on the circle.

Examples $\overline{AB}$ and $\overline{DE}$ are chords of $\odot C$.

A **diameter** of a circle is a chord that passes through the center and is made up of collinear radii.

Example $\overline{DE}$ is a diameter of $\odot C$. Diameter $\overline{DE}$ is made up of collinear radii $\overline{CD}$ and $\overline{CE}$.



KeyConcept Radius and Diameter Relationships

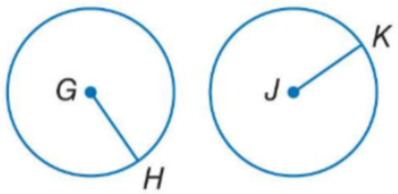
If a circle has radius r and diameter d, the following relationships are true.

Radius Formula $r = \frac{d}{2}$ or $r = \frac{1}{2}d$

Diameter Formula $d = 2r$

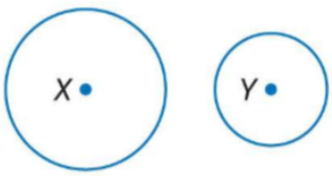
KeyConcept Circle Pairs

Two circles are congruent if and only if they have congruent radii.



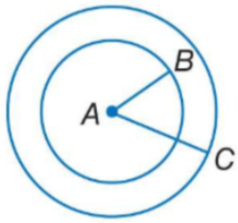
Example $\overline{GH} \cong \overline{JK}$, so $\odot G \cong \odot J$.

All circles are similar.



Example $\odot X \sim \odot Y$

Concentric circles are coplanar circles that have the same center.



Example $\odot A$ with radius $\overline{AB}$ and $\odot A$ with radius $\overline{AC}$ are concentric.

Two circles can intersect in two different ways.

2 Points of Intersection	1 Point of Intersection	No Points of Intersection
		

KeyConcept Circumference

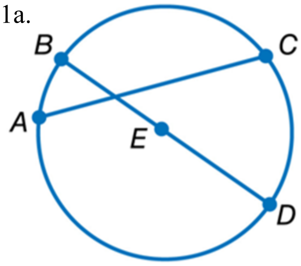
Words If a circle has diameter d or radius r, the circumference C equals the diameter times pi or twice the radius times pi.

Symbols $C = \pi d$ or $C = 2\pi r$

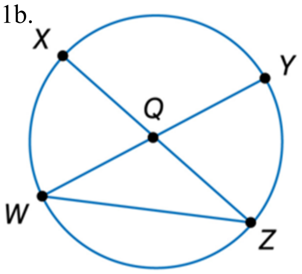
Geo C9 handouts to put in notes

Section 9.1

1a.



1b.



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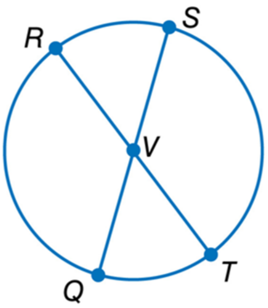

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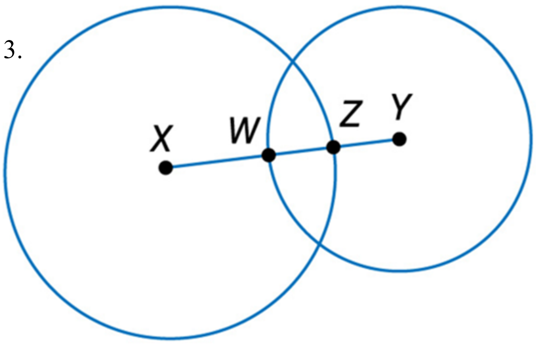
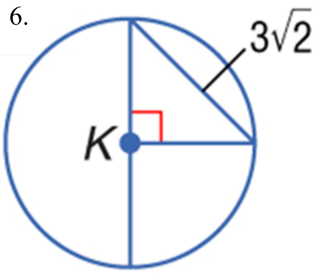


4.
CROP CIRCLES A series of crop circles was discovered in Alberta, Canada, on September 4, 1999. The largest of the three circles had a radius of 30 feet. Find its circumference.

5.
Find the diameter and the radius of a circle to the nearest hundredth if the circumference of the circle is 65.4 feet.

6.

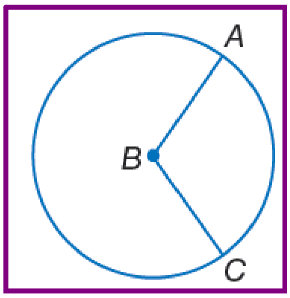


Geo C9 handouts to put in notes

Section 9.2

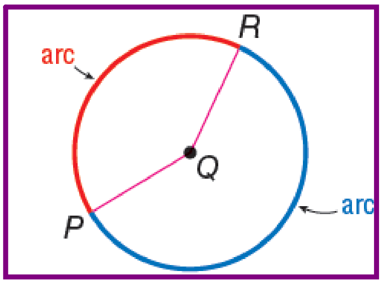
KEY TERMS

Angles and Arcs A **central angle** of a circle is an angle with a vertex in the center of the circle. Its sides contain two radii of the circle. $\angle ABC$ is a central angle of $\odot B$.



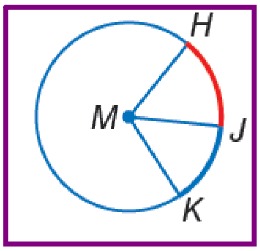
Recall from Lesson 1-4 that a *degree* is $\frac{1}{360}$ of the circular rotation about a point.

An **arc** is a portion of a circle defined by two endpoints. A central angle separates the circle into two arcs with measures related to the measure of the central angle.



Congruent arcs are arcs in the same or congruent circles that have the same measure.

Adjacent arcs are arcs in a circle that have exactly one point in common. In $\odot M$, $\widehat{HJ}$ and $\widehat{JK}$ are adjacent arcs. As with adjacent angles, you can add the measures of adjacent arcs.



Arc Length **Arc length** is the distance between the endpoints along an arc measured in linear units. Since an arc is a portion of a circle, its length is a fraction of the circumference.

StudyTip

Naming Arcs Minor arcs are named by their endpoints. Major arcs and semicircles are named by their endpoints and another point on the arc that lies between these endpoints.

Terminology The terms arc measure and arc length are not interchangeable. Like angles, arcs have degree measure, denoted $m\widehat{AC}$. Just as with segment length arc length is a distance along a curve.

Watch Out!

Arc Length The length of an arc is given in linear units, such as centimeters. The measure of an arc is given in degrees.

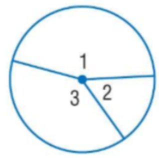
Geo C9 handouts to put in notes

KeyConcept Sum of Central Angles

Section 9.2

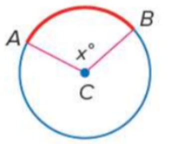
Words The sum of the measures of the central angles of a circle with no interior points in common is 360.

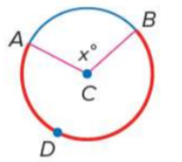
Example $m\angle 1 + m\angle 2 + m\angle 3 = 360$

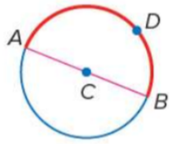


Key Concept Arcs and Arc Measure

Arc	Measure
A **minor arc** is the shortest arc connecting two endpoints on a circle.	The measure of a minor arc is less than 180 and equal to the measure of its related central angle. $m\widehat{AB} = m\angle ACB = x$
A **major arc** is the longest arc connecting two endpoints on a circle.	The measure of a major arc is greater than 180, and equal to 360 minus the measure of the minor arc with the same endpoints. $m\widehat{ADB} = 360 - m\widehat{AB} = 360 - x$
A **semicircle** is an arc with endpoints that lie on a diameter.	The measure of a semicircle is 180. $m\widehat{ADB} = 180$



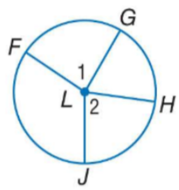




Theorem 10.1

Words In the same circle or in congruent circles, two minor arcs are congruent if and only if their central angles are congruent.

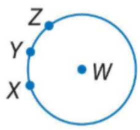
Example If $\angle 1 \cong \angle 2$, then $\widehat{FG} \cong \widehat{HJ}$.
If $\widehat{FG} \cong \widehat{HJ}$, then $\angle 1 \cong \angle 2$.



Postulate 10.1 Arc Addition Postulate

Words The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Example $m\widehat{XYZ} = m\widehat{XY} + m\widehat{YZ}$

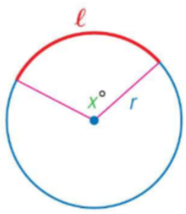


KeyConcept Arc Length

Words The ratio of the length of an arc ℓ to the circumference of the circle is equal to the ratio of the degree measure of the arc to 360.

Proportion $\frac{\ell}{2\pi r} = \frac{x}{360}$ or

Equation $\ell = \frac{x}{360} \cdot 2\pi r$



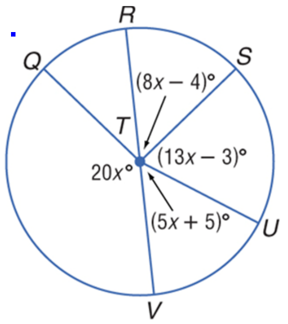
$$\theta = \frac{\ell}{r}$$

Arc length equation, when angle θ is given in radians.

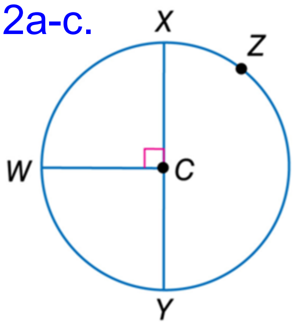
Geo C9 handouts to put in notes

Section 9.2

1.



2a-c.



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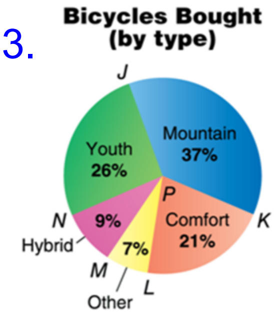
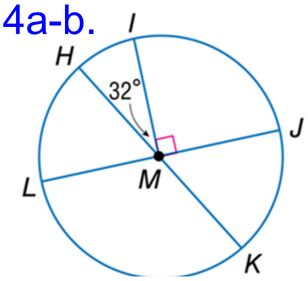


a. **Find $m\widehat{KL}$.**

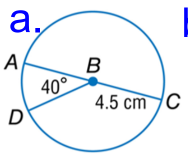
b. **Find $m\widehat{NJL}$.**

4a-b.

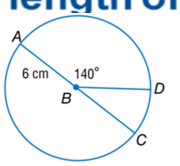


5. **Find the length of $\widehat{DA}$.**

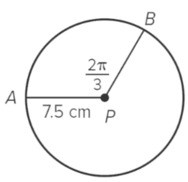
a.



b.



6. **Find the length of $\widehat{AB}$.**



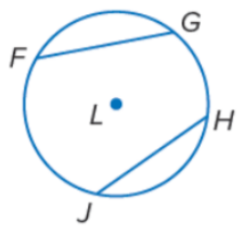
Geo C9 handouts to put in notes

Theorem 9.2

Section 9.3

Words In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Example $\widehat{FG} \cong \widehat{HJ}$ if and only if $\overline{FG} \cong \overline{HJ}$.



Proof Theorem 9.2 (part 1)

Given: $\odot P$; $\widehat{QR} \cong \widehat{ST}$

Prove: $\overline{QR} \cong \overline{ST}$

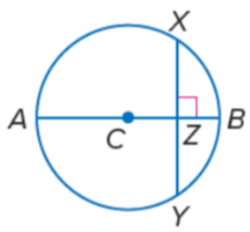
Proof:

Statements	Reasons
1. $\odot P$, $\widehat{QR} \cong \widehat{ST}$	1.
2. $\angle QPR \cong \angle SPT$	2. If arcs are ≅, their corresponding central ∠s are ≅.
3. $\overline{QP} \cong \overline{PR} \cong \overline{SP} \cong \overline{PT}$	3. All radii of a circle are ≅.
4. $\triangle PQR \cong \triangle PST$	4.
5. $\overline{QR} \cong \overline{ST}$	5.

Theorems

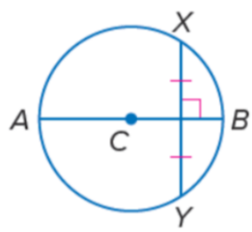
9.3 If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.

Example If diameter $\overline{AB}$ is perpendicular to chord $\overline{XY}$, then $\overline{XZ} \cong \overline{ZY}$ and $\widehat{XB} \cong \widehat{BY}$.



9.4 The perpendicular bisector of a chord is a diameter (or radius) of the circle.

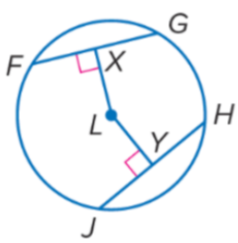
Example If $\overline{AB}$ is a perpendicular bisector of chord $\overline{XY}$, then $\overline{AB}$ is a diameter of $\odot C$.



Theorem 9.5

Words In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

Example $\overline{FG} \cong \overline{JH}$ if and only if $LX = LY$.



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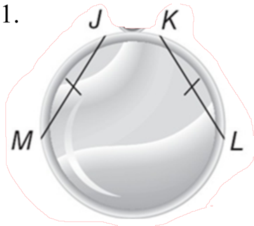
Section 9.3

1.



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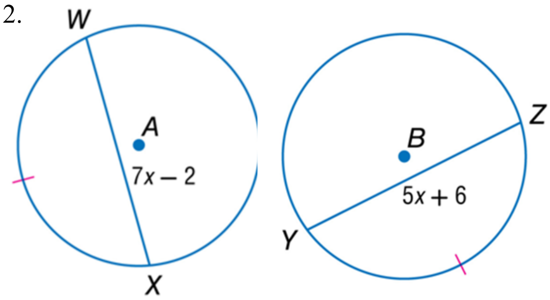

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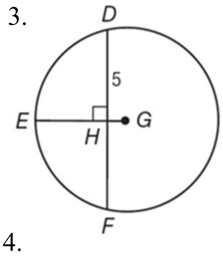

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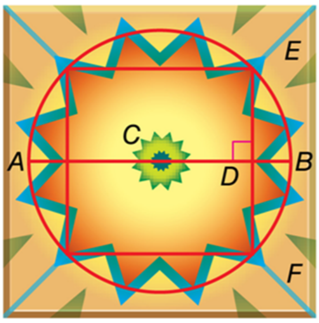

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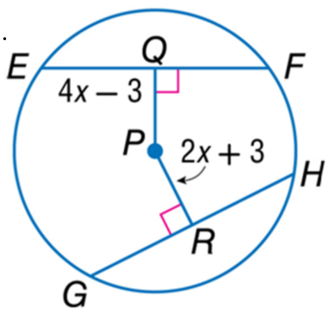
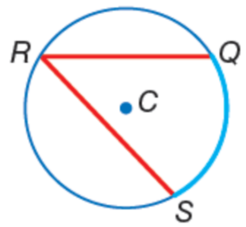


Geo C9 handouts to put in notes

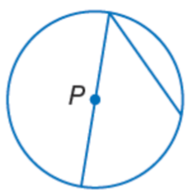
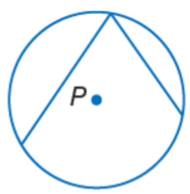
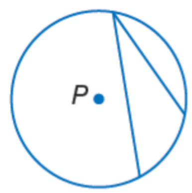
Section 9.4

KEY TERMS



Inscribed Angle - An angle that has a vertex on a circle and sides that contain chords of a circle. In circle C, ∠QRS is an inscribed angle.

Intercepted Arc - An arc with endpoints on the sides of an inscribed angle and lies in the interior of the inscribed angle. In circle C, minor arc QS is intercepted by ∠QRS.

Case 1	Case 2	Case 3
		
Center *P* is on a side of the inscribed angle.	Center *P* is inside the inscribed angle.	The center *P* is in the exterior of the inscribed angle.

Vocabulary Link

inscribed

Everyday Use: written on or in a surface, such as inscribing the inside of a ring with an inscription

Math Use: touching only the sides (or interior) of another figure

StudyTip

Inscribed Polygons Remember that for a polygon to be an inscribed polygon, *all* of its vertices must lie on the circle.

StudyTip

Quadrilaterals Theorem 10.9 can be verified by considering that the arcs intercepted by opposite angles of an inscribed quadrilateral form a circle.

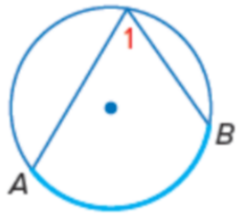
Geo C9 handouts to put in notes

Theorem 9.6 Inscribed Angle Theorem

Section 9.4

Words If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

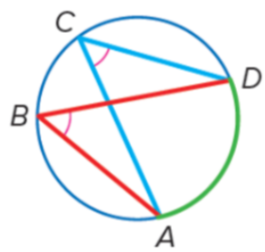
Example $m\angle 1 = \frac{1}{2} m\widehat{AB}$ and $m\widehat{AB} = 2m\angle 1$



Theorem 9.7

Words If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent.

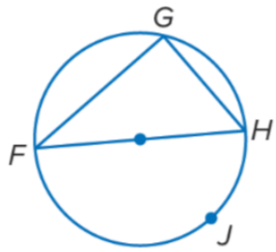
Example $\angle B$ and $\angle C$ both intercept $\widehat{AD}$. So, $\angle B \cong \angle C$.



Theorem 9.8

Words An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle.

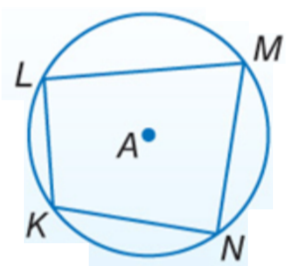
Example If $\widehat{FJH}$ is a semicircle, then $m\angle G = 90$. If $m\angle G = 90$, then $\widehat{FJH}$ is a semicircle and $\overline{FH}$ is a diameter.



Theorem 9.9

Words If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

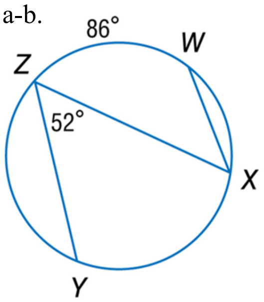
Example If quadrilateral $KLMN$ is inscribed in $\odot A$, then $\angle L$ and $\angle N$ are supplementary and $\angle K$ and $\angle M$ are supplementary.



Geo C9 handouts to put in notes

Section 9.4

1a-b.



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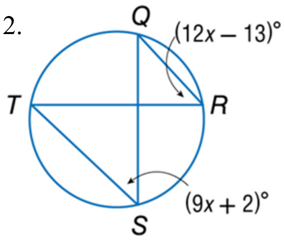
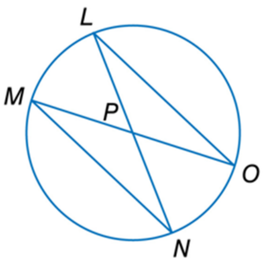


3.

Given: $\overparen{LO} \cong \overparen{MN}$

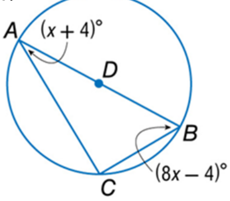
Prove: $\Delta MNP \cong \Delta LOP$



Proof:

Statements	Reasons

4.



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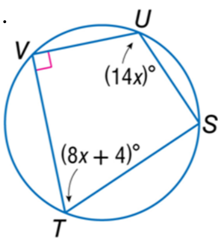
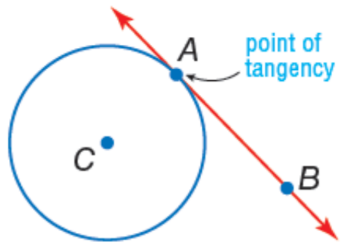


Geo C9 handouts to put in notes

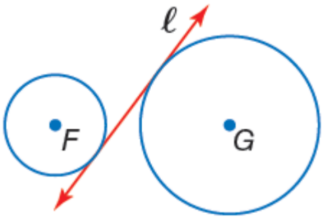
Section 9.5

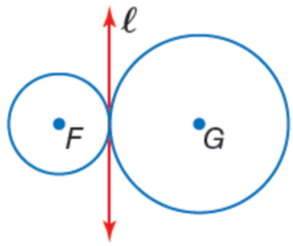
KEY TERMS



Tangent -A line in the same plane as a circle that intersects the circle in exactly one point, called the point of tangency. AB is tangent to circle C at point A. AB and AB are also called tangents.

Common Tangent - A line, ray, or segment that is tangent to two circles in the same plane. In each figure below, line ℓ is a common tangent to circles F and G.

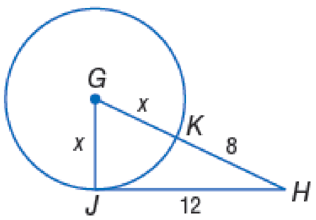




The shortest distance from a tangent to the center of a circle is the radius drawn to the point of tangency.

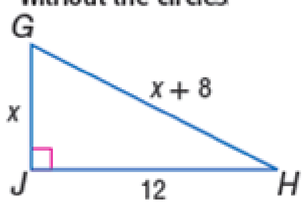
Circumscribed Polygon - A polygon is circumscribed about a circle if every side of the polygon is tangent to the circle.

Circumscribed Polygons	Polygons Not Circumscribed
	



Problem-SolvingTip

Solve a Simpler Problem You can use the *solve a simpler problem* strategy by sketching and labeling the right triangles without the circles



Watch Out!

Identifying Circumscribed Polygons Just because the circle is tangent to one or more of the sides of a polygon does not mean that the polygon is circumscribed about the circle, as shown in the second set of figures.

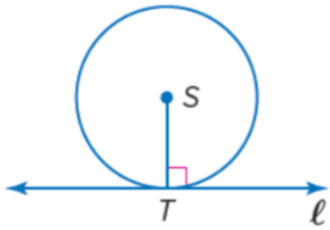
Geo C9 handouts to put in notes

Section 9.5

Theorem 9.10

Words In a plane, a line is tangent to a circle if and only if it is perpendicular to a radius drawn to the point of tangency.

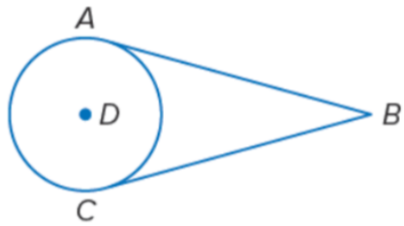
Example Line ℓ is tangent to $\odot S$ if and only if $\ell \perp \overline{ST}$.



Theorem 9.11

Words If two segments from the same exterior point are tangent to a circle, then they are congruent.

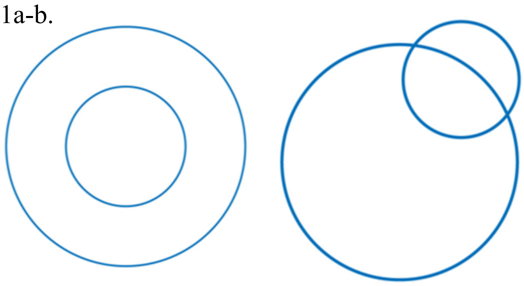
Example If $\overline{AB}$ and $\overline{CB}$ are tangent to $\odot D$, then $\overline{AB} \cong \overline{CB}$.



Geo C9 handouts to put in notes

Section 9.5

1a-b.



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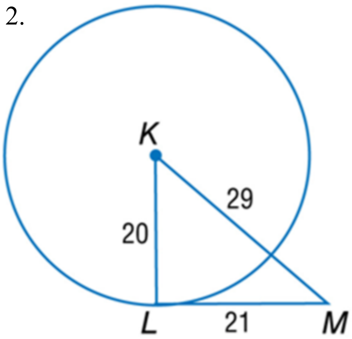

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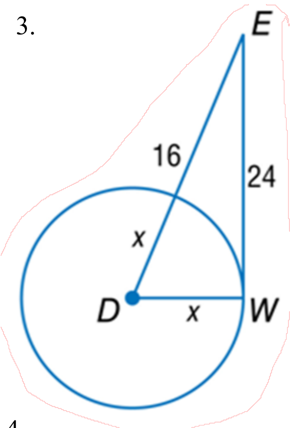

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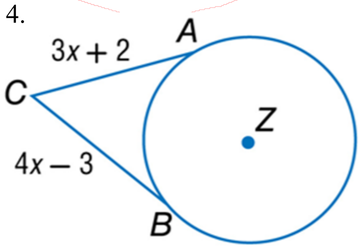

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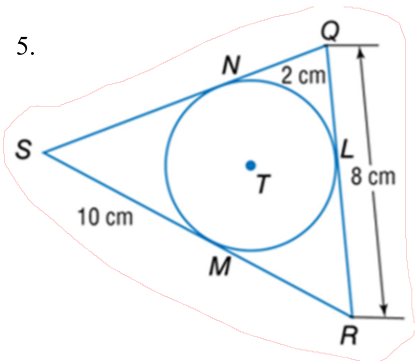
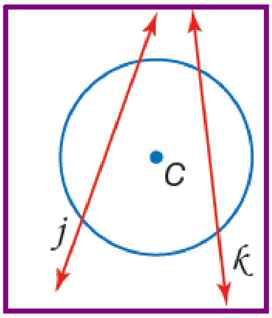


Section 9.6

KEY TERMS

Intersections On or Inside a Circle A **secant** is a line that intersects a circle in exactly two points. Lines j and k are secants of $\odot C$.



When two secants intersect inside a circle, the angles formed are related to the arcs they intercept.

Intersections Outside a Circle Secants and tangents can also meet outside a circle. The measure of the angle formed also involves half of the measures of the arcs they intercept.

StudyTip

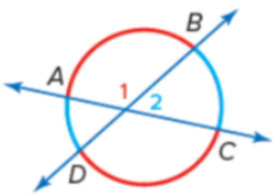
Absolute Value The measure of each $\angle A$ can also be expressed as half the absolute value of the difference of the arc measure. In this way, the order of the arc measures does not affect the outcome of the calculation.

Geo C9 handouts to put in notes

Theorem 9-12

Section 9.6

Words If two secants or chords intersect in the interior of a circle, then the measure of an angle formed is one half the *sum* of the measure of the arcs intercepted by the angle and its vertical angle.



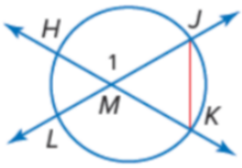
Example $m\angle 1 = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$ and $m\angle 2 = \frac{1}{2}(m\widehat{DA} + m\widehat{BC})$

Proof

Given: $\overleftrightarrow{HK}$ and $\overleftrightarrow{JL}$ intersect at M.

Prove: $m\angle 1 = \frac{1}{2}(m\widehat{JH} + m\widehat{LK})$

Proof:

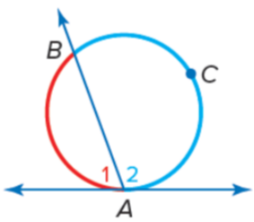


Statements	Reasons
1. $\overleftrightarrow{HK}$ and $\overleftrightarrow{JL}$ intersect at M.	1. Given
2. $m\angle 1 = m\angle MJK + m\angle MKJ$	2. Exterior Angle Theorem
3. $m\angle MJK = \frac{1}{2}m\widehat{LK}$, $m\angle MKJ = \frac{1}{2}m\widehat{JH}$	3. The measure of an inscribed $\angle$ equals half the measure of the intercepted arc.
4. $m\angle 1 = \frac{1}{2}m\widehat{LK} + \frac{1}{2}m\widehat{JH}$	4. Substitution
5. $m\angle 1 = \frac{1}{2}(m\widehat{JH} + m\widehat{LK})$	5. Distributive Property

Theorem 9-13

Words If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its intercepted arc.

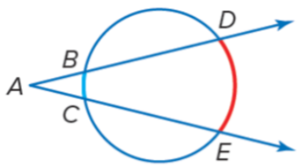
Example $m\angle 1 = \frac{1}{2}m\widehat{AB}$ and $m\angle 2 = \frac{1}{2}m\widehat{ACB}$



Theorem 9.14

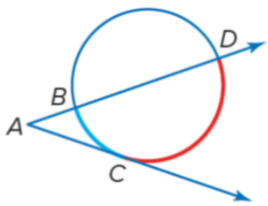
Words If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.

Examples



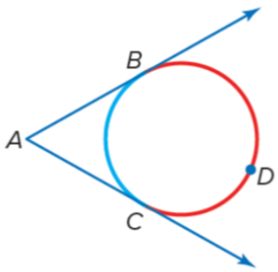
Two Secants

$m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$



Secant-Tangent

$m\angle A = \frac{1}{2}(m\widehat{DC} - m\widehat{BC})$

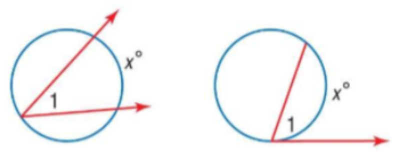
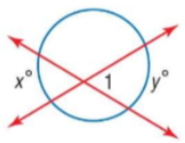
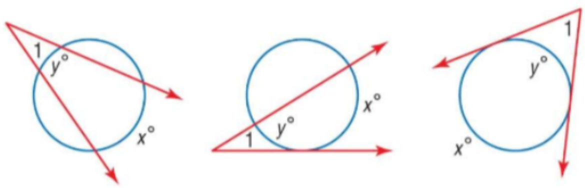


Two Tangents

$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$

Geo C9 handouts to put in notes

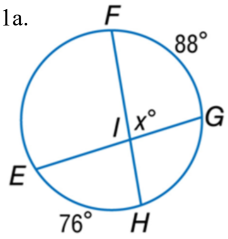
KeyConcept Circle and Angle Relationships — Section 9.6

Vertex of Angle	Model(s)	Angle Measure
on the circle		one half the measure of the intercepted arc $m\angle 1 = \frac{1}{2}x$
inside the circle		one half the measure of the sum of the intercepted arc $m\angle 1 = \frac{1}{2}(x + y)$
outside the circle		one half the measure of the difference of the intercepted arcs $m\angle 1 = \frac{1}{2}(x - y)$

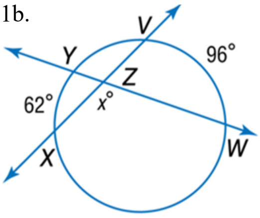
Geo C9 handouts to put in notes

Section 9.6

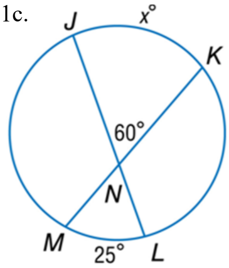
1a.



1b.



1c.



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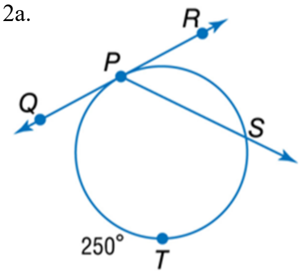
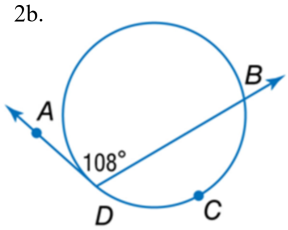
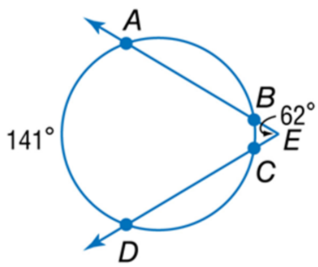


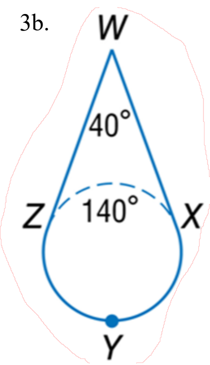
2b.



3a.



3b.



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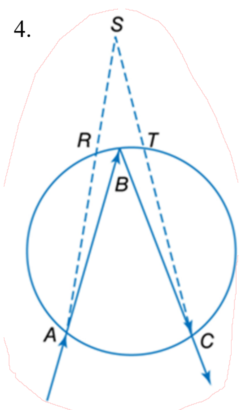
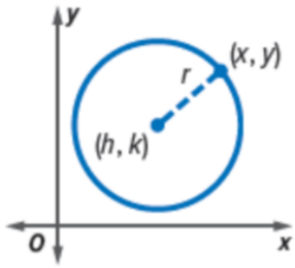


Geo C9 handouts to put in notes

Key Concept Equation of a Circle in Standard Form — Section 9.7

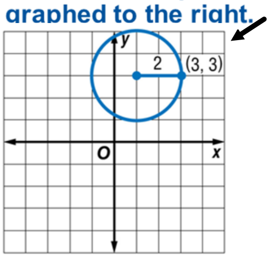
The standard form of the equation of a circle with center at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

The standard form of the equation of a circle is also called the *center-radius* form.



1A. Write the equation of the circle with a center at (3, –3) and a radius of 6.

1B. Write the equation of the circle graphed to the right.



2. Write the equation of the circle with center at (–3, –2), and passes through (1, –2).

3. The equation of a circle is $x^2 - 4x + y^2 + 6y = -9$. State the coordinates of the center and the measure of the radius. Then graph the equation.