Section 0.9

A radical expression is an expression that contains a square root. The expression is in simplest form when the following three conditions have been met.

- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

The **Product Property** states that for two numbers a and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

For radical expressions in which the exponent of the variable inside the radical is *even* and the resulting simplified exponent is *odd*, you must use absolute value to ensure nonnegative results.

The **Quotient Property** states that for any numbers a and b, where $a \geq 0$ and $b \geq 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Rationalizing the denominator of a radical expression is a method used to eliminate radicals from the denominator of a fraction. To rationalize the denominator, multiply the expression by a fraction equivalent to 1 such that the resulting denominator is a perfect square.

Sometimes, conjugates are used to simplify radical expressions. Conjugates are binomials of the form $p\sqrt{q} + r\sqrt{t}$ and $p\sqrt{q} - r\sqrt{t}$.

Geo C8 handouts to put in notes

KeyConcept Geometric Mean

Section 8.1

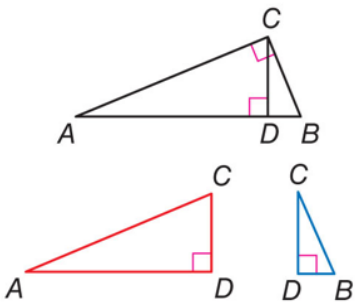
Words The geometric mean of two positive numbers a and b is the number x such that $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

Example The geometric mean of $a = 9$ and $b = 4$ is 6, because $6 = \sqrt{9 \cdot 4}$.

Theorem 8.1

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

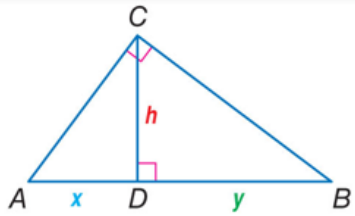
Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\triangle ACD \sim \triangle ABC$, $\triangle CBD \sim \triangle ABC$, and $\triangle ACD \sim \triangle CBD$.



Theorems Right Triangle Geometric Mean Theorems

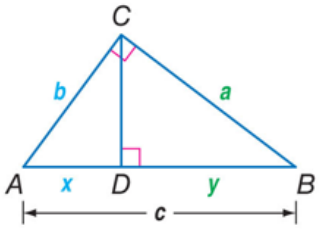
8.2 Geometric Mean (Altitude) Theorem The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\frac{x}{h} = \frac{h}{y}$ or $h = \sqrt{xy}$.

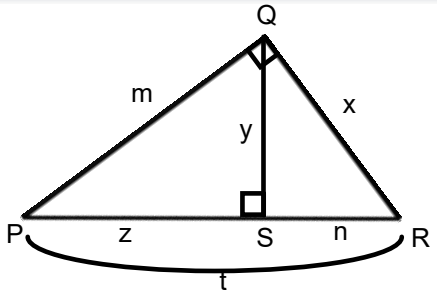


8.3 Geometric Mean (Leg) Theorem The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Example If $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$ of right $\triangle ABC$, then $\frac{c}{b} = \frac{b}{x}$ or $b = \sqrt{xc}$ and $\frac{c}{a} = \frac{a}{y}$ or $a = \sqrt{yc}$.



m =



x =

y =

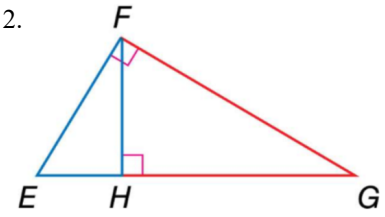
Remember:
t = z + n
so
z =
n =

Geo C8 handouts to put in notes

Section 8.1

1. **Find the geometric mean between 2 and 50.**

2.



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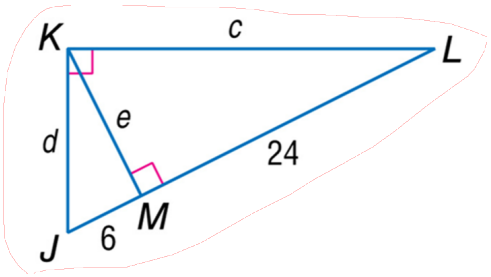

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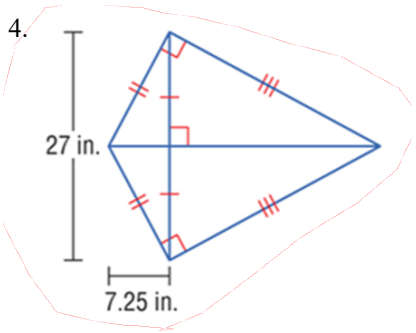
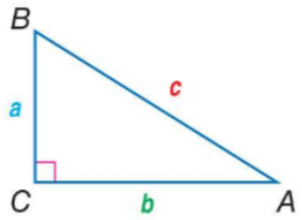


Section 8.2

Theorem 8.4 Pythagorean Theorem

Words In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Symbols If $\triangle ABC$ is a right triangle with right angle C, then $a^2 + b^2 = c^2$.



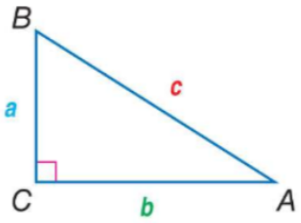
KeyConcept Common Pythagorean Triples

3, 4, 5	5, 12, 13	8, 15, 17	7, 24, 25
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

Theorem 8.5 Converse of the Pythagorean Theorem

Words If the sum of the squares of the lengths of the shortest sides of a triangle is equal to the square of the length of the longest side, then the triangle is a right triangle.

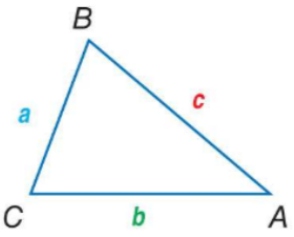
Symbols If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.



Theorems Pythagorean Inequality Theorems

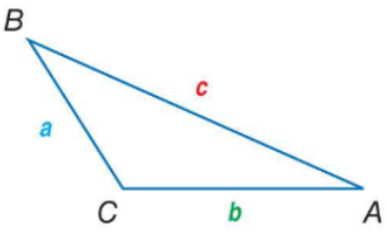
8.6 If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is an acute triangle.

Symbols If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



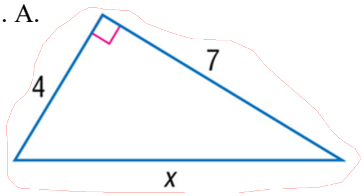
8.7 If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is an obtuse triangle.

Symbols If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.

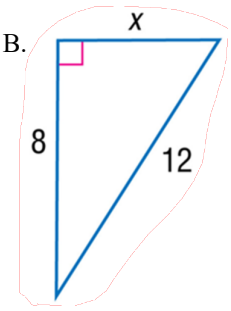


Section 8.2

1. A.



1. B.



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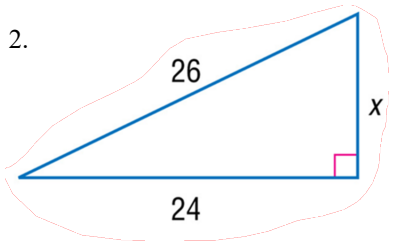

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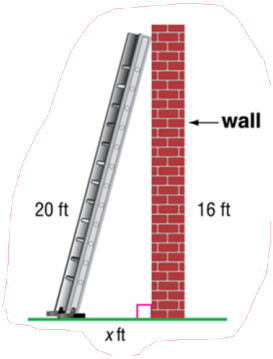


4. A. Lengths 9, 12, 15. Triangle?_____
If so, acute, right or obtuse? _____

4. B. Lengths 10, 11, 13. Triangle?_____
If so, acute, right or obtuse? _____

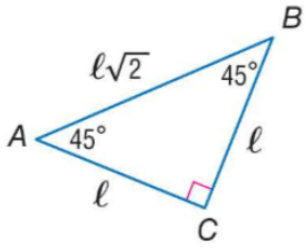
Geo C8 handouts to put in notes

Theorem 8.8 45°-45°-90° Triangle Theorem

Section 8.3

In a 45°-45°-90° triangle, the legs ℓ are congruent and the length of the hypotenuse h is $\sqrt{2}$ times the length of a leg.

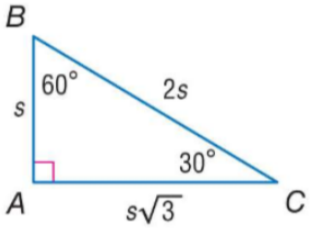
Symbols In a 45°-45°-90° triangle, $\ell = \ell$ and $h = \ell\sqrt{2}$.

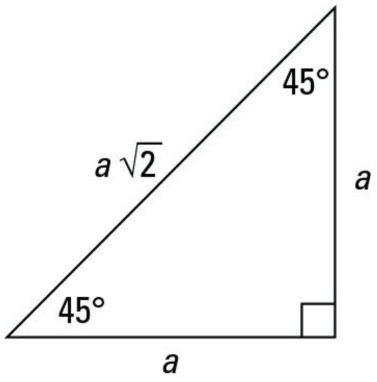


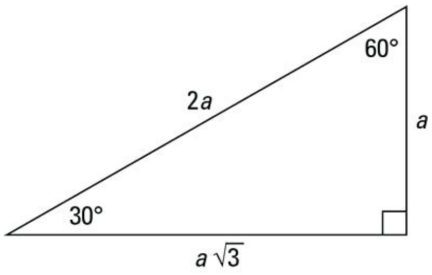
Theorem 8.9 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse h is 2 times the length of the shorter leg s, and the length of the longer leg ℓ is $\sqrt{3}$ times the length of the shorter leg.

Symbols In a 30°-60°-90° triangle, $h = 2s$ and $\ell = s\sqrt{3}$.



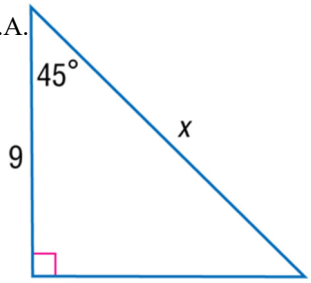




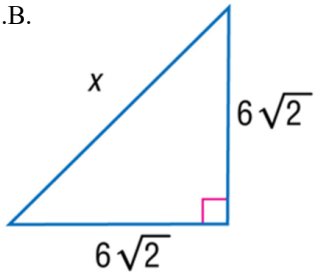
Geo C8 handouts to put in notes

Section 8.3

1.A.



1.B.



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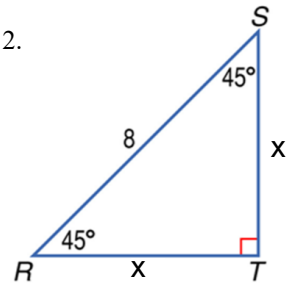

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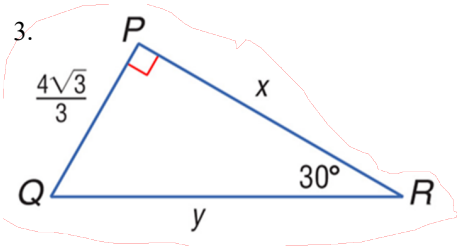

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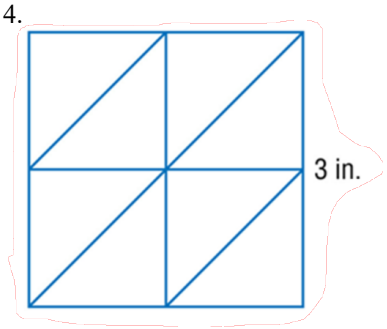
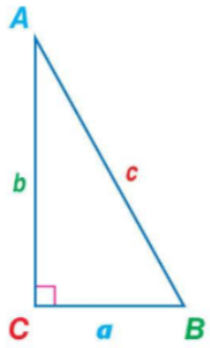


Geo C8 handouts to put in notes

KeyConcept Trigonometric Ratios

Section 8.4

Words	Symbols
If $\triangle ABC$ is a right triangle with acute $\angle A$, then the **sine** of $\angle A$ (written sin A) is the ratio of the length of the leg opposite $\angle A$ (opp) to the length of the hypotenuse (hyp).	$\sin A = \frac{\text{opp}}{\text{hyp}}$ or $\frac{a}{c}$ $\sin B = \frac{\text{opp}}{\text{hyp}}$ or $\frac{b}{c}$
If $\triangle ABC$ is a right triangle with acute $\angle A$, then the **cosine** of $\angle A$ (written cos A) is the ratio of the length of the leg adjacent $\angle A$ (adj) to the length of the hypotenuse (hyp).	$\cos A = \frac{\text{adj}}{\text{hyp}}$ or $\frac{b}{c}$ $\cos B = \frac{\text{adj}}{\text{hyp}}$ or $\frac{a}{c}$
If $\triangle ABC$ is a right triangle with acute $\angle A$, then the **tangent** of $\angle A$ (written tan A) is the ratio of the length of the leg opposite $\angle A$ (opp) to the length of the leg adjacent $\angle A$ (adj).	$\tan A = \frac{\text{opp}}{\text{adj}}$ or $\frac{a}{b}$ $\tan B = \frac{\text{opp}}{\text{adj}}$ or $\frac{b}{a}$



KeyConcept Inverse Trigonometric Ratios

Words	If $\angle A$ is an acute angle and the sine of A is x, then the **inverse sine** of x is the measure of $\angle A$.
Symbols	If $\sin A = x$, then $\sin^{-1} x = m\angle A$.
Words	If $\angle A$ is an acute angle and the cosine of A is x, then the **inverse cosine** of x is the measure of $\angle A$.
Symbols	If $\cos A = x$, then $\cos^{-1} x = m\angle A$.
Words	If $\angle A$ is an acute angle and the tangent of A is x, then the **inverse tangent** of x is the measure of $\angle A$.
Symbols	If $\tan A = x$, then $\tan^{-1} x = m\angle A$.

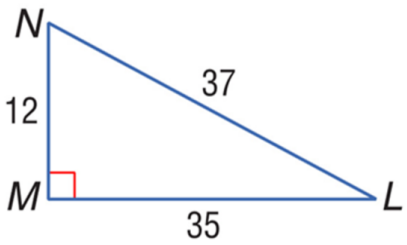
Section 8.4

Sentence Frame: Learning SOHCAHTOA

The_____ of an angle is the ratio of
_____ over _____.

Section 8.4

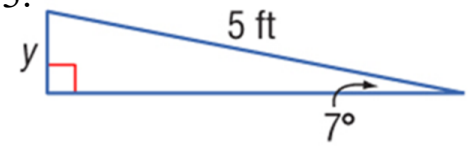
1 a-f.



2.

Use a special right triangle to express the cosine of 60° as a fraction and as a decimal to the nearest hundredth.

3.



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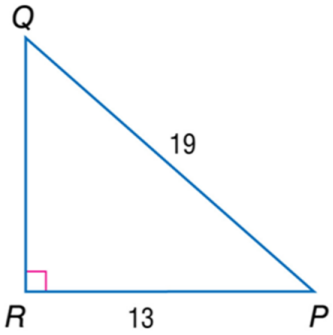

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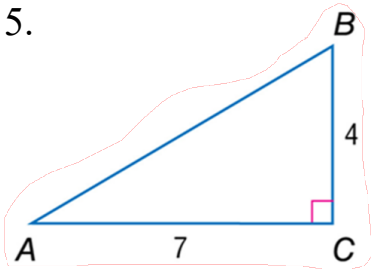
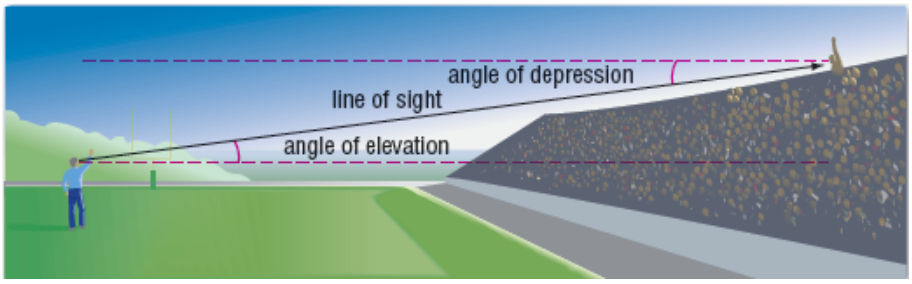


Section 8.5

Angle of Elevation -An an angle formed by a **horizontal** line and an observer's line of sight to an object *above* the horizontal line.

Angle of Depression -An an angle formed by a **horizontal** line and an observer's line of sight to an object *below* the horizontal line.

Angles of elevation and depression are ALWAYS formed with at horizontal line and never a vertical line.



Horizontal lines are parallel, so the angle of elevation and angle of depression in the diagram are congruent by the Alternate Interior Angles Theorem.

Two Angles of Elevation or Depression Angles of elevation or depression to two different objects can be used to estimate the distance between those objects. Similarly, the angles from two different positions of observation to the same object can be used to estimate the object's height.

StudyTip

Indirect Measurement
When using the angles of depression to two different objects to calculate the distance between them, it is important to remember that the two objects must lie in the same horizontal plane. In other words, one object cannot be higher or lower than the other.

Geo C8 handouts to put in notes

Section 8.5

1. At the circus, a person in the auditence at ground level watches the high-wire routine. A 5-foot-6-inch tall acrobat is standing on a platform that is 25 feet off the ground. How far is the audience member from the base of the platform, if the angle of elevation from the audience member's line of sight to the top of the acrobat's head is 27°?

2. Maria is at the top of a cliff and sees a seal in the water. If the cliff is 40 feet above the water and the angle of depression is 52°, what is the horizontal distance from the seal to the cliff, to the nearest foot?

3. Vernon is on the top deck of a cruise ship and observes two dolphins following each other directly away from the ship in a straight line. Vernon's position is 154 meters above sea level, and the angles of depression to the two dolphins are 35° and 36°. Find the distance between the two dolphins to the nearest meter?

•

Geo C8 handouts to put in notes

KeyConcept Area of a Triangle

Section 8.6

Words The area of a triangle is one half the product of the lengths of two sides and the sine of their included angle.

Symbols $\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$