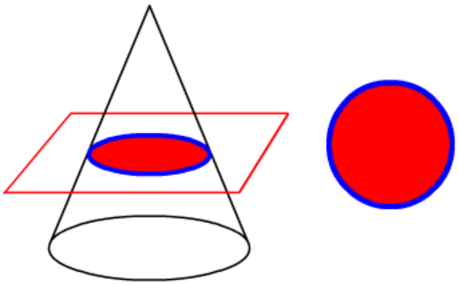
http://www.coolmath.com/algebra/25-conic-sections/01-introduction-circles-01

Circles - Intro

So, what's the deal with conic sections?

All the shapes we'll be talking about in this chapter are slices out of a cone.

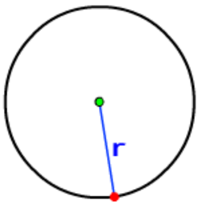
If you slice a cone horizontally, you get a circle:



What makes a circle a circle?

Geometrically, that is... Yes, I know circles are round, but, let's get a little more specific.

Check out this animation:
A line segment of a set length sweeps out a circle.

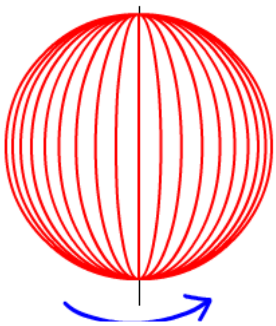


A little more technically, all those red points on the circle are the same distance from the green center point.

What if you took a long rope and a board or plank... Can you think of a way to make a crop circle?

Check out this link: CircleMakers.org

Yeah, circles are cool things -- you see them everywhere. But, spheres are really where it's at!

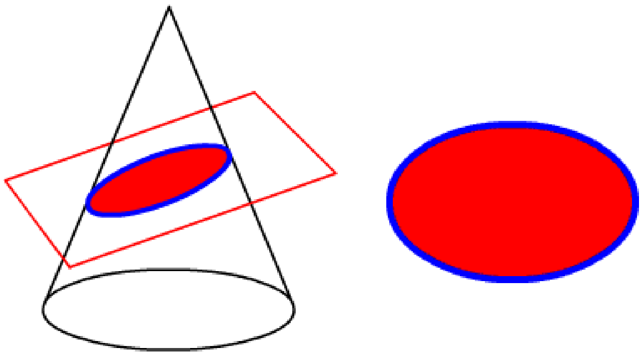


Slices of spheres are used for all sorts of lenses in optics.

This Fresnel lamp uses slices of spheres to help aim light.

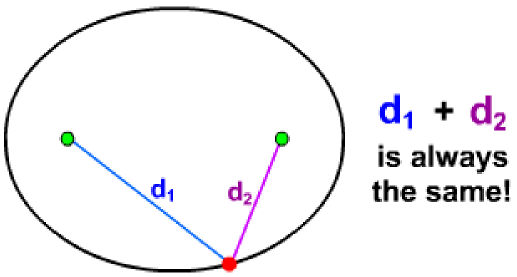
Ellipses - Intro

If you slice a cone at a diagonal, you get an ellipse:

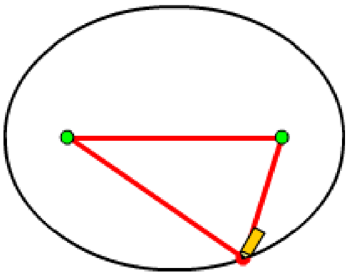


Here's what geometrically makes an ellipse an ellipse:

You start with the two green dots...



So, if you put two nails on a board and put a loop of string around them...

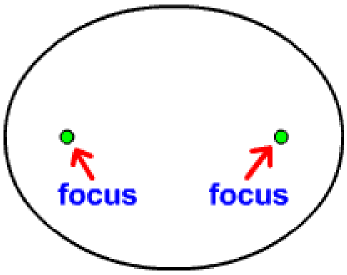


You can draw a perfect ellipse! Just keep the string pulled tightly.

Think about it... That loop of string will always be the same length -- won't it?

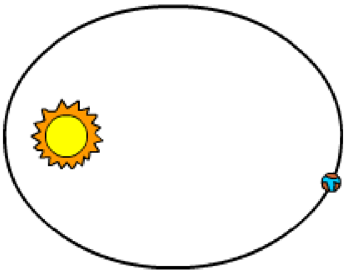
That's what makes an ellipse!

By the way, those green dots are called foci (the plural for focus -- it's one of those pretentious math words that make geeks feel superior.)



These points are pretty dang important!

In fact, our Sun lives at a focus... All the planets orbit the Sun in elliptical paths.



Notice how the Earth speeds up a bit when it gets closer to the Sun? Cooooooool! (Actually, it wouldn't be since you're closer to the Sun.)

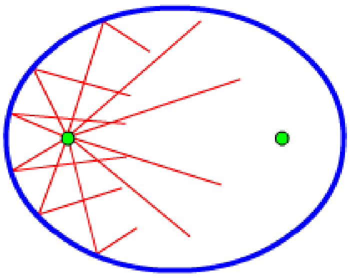
(I exagerated this animation quite a bit too make my point. Although our orbits are ellipses, they are much closer to being circles.)

I know you're pretty dang jazzed about ellipses already (and, really, who can blame you), but, there's more!

These things have some totally amazing reflective properties!

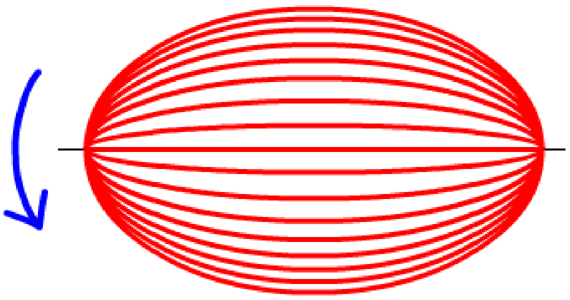
Check it out:

If we shoot rays (like maybe light or sound) from one focus...



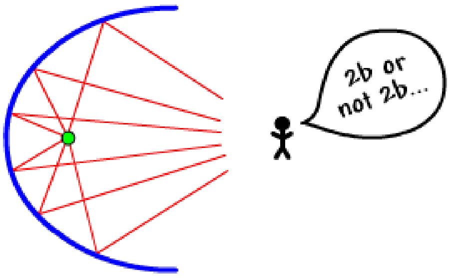
They shoot out, reflect off the inside of the ellipse and head straight for the other focus!
Repeat after me: Duuuuuuuuuuude!

This is pretty useful too -- especially with ellipsoids (3-D ellipses).



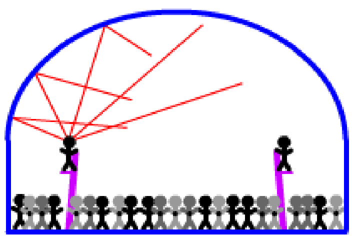
Here's an ellipsoidal reflector spotlight.

You put the light bulb at one focus... and an actor at the other!



And what about reflecting sound?

Some science museums have what's called "whisper rooms." The ceiling is an ellipsoid...

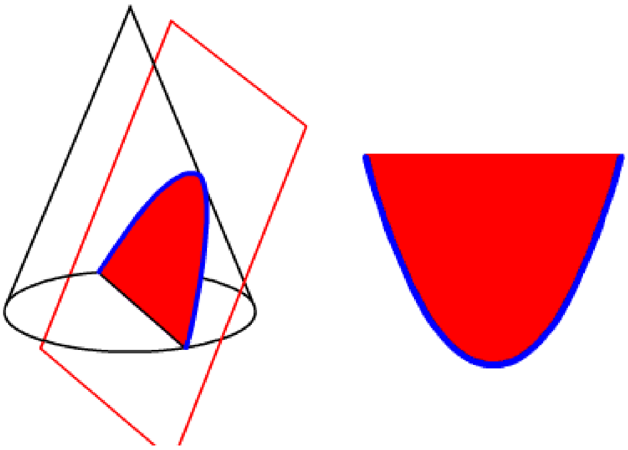


If you stand at one focus... and whisper... someone standing at the other focus will be able to hear you over a noisy crowd!

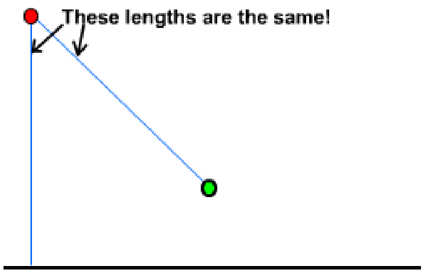
Parabolas - Intro

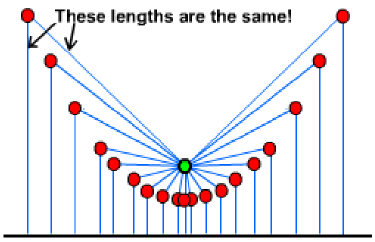
Ya know what you get if you slice a cone parallel to the edge?

Our old friend the parabola!



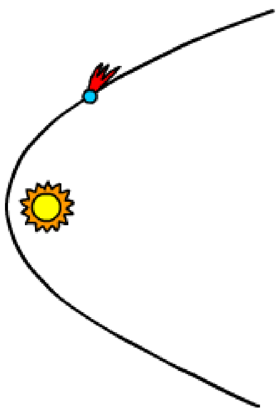
Here's what makes the parabola special -- geometry-wise:





The green dot is a focus.

Just like with ellipses, the focus is a special place...



Some comets shoot through our solar system along parabolic paths with the Sun at the focus.
Notice how the comet speeds up as it gets closer to the Sun... It's a gravity thing!

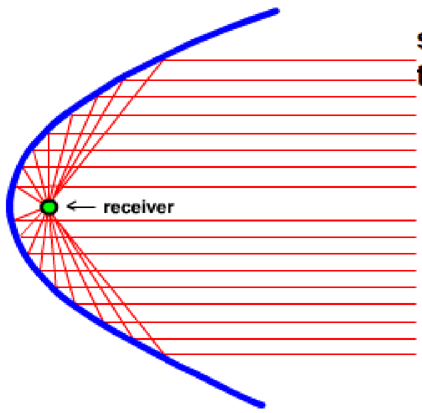
(And, no, I didn't screw up on the comet's tail. The solar wind always blows the tail away from the Sun. By the way, I highly recommend that you take an astronomy class in college. You'll love it!)

Back to parabolas...

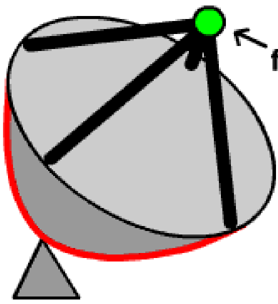
Remember the parabolas we saw before? In bridges... and paths of projectiles like when you throw a ball in the air.

They have cool reflective properties too.

Check it out:



This is why paraboloids are used as satellite dishes and as the mirrors in telescopes.

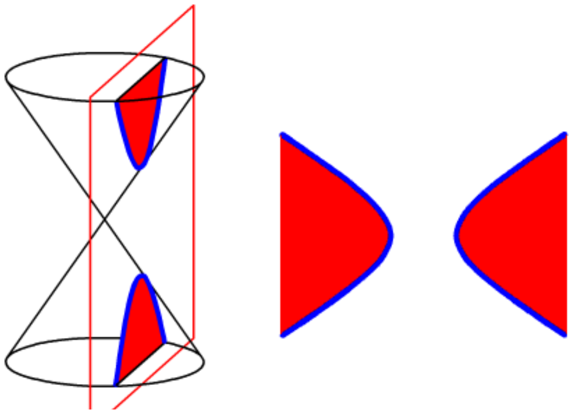


And check out these flood lights:

Can you see from the reflective properties who paraboloids would make great flood lights? (Compare this to the ellipsoidal spot lights.)

Hyperbolas - Intro

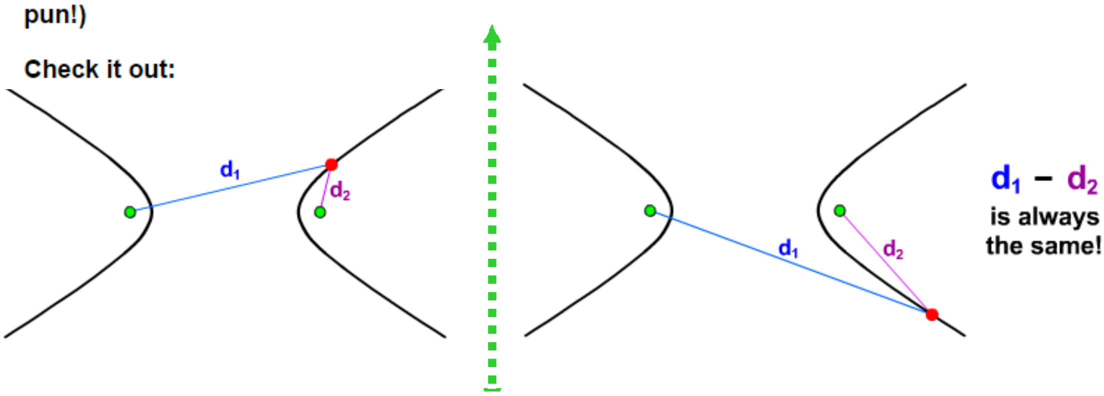
If you slice two cones vertically, you get a hyperbola:



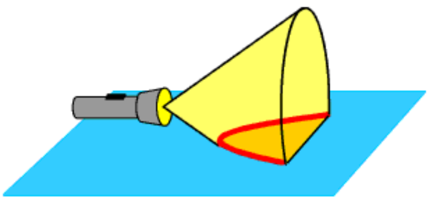
Here's what geometrically makes a hyperbola a hyperbola:

Just like the ellipse, you start with the two green dots (the foci)... B
there's a big **DIFFERENCE** with hyperbolas! (Ha! I just made a mat
pun!)

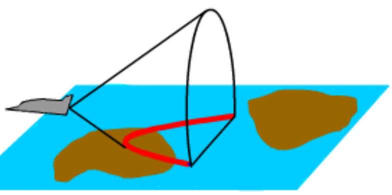
Check it out:



If you shine a flashlight on a table horizontally, you'll get a hyperbola -- well, half anyway:

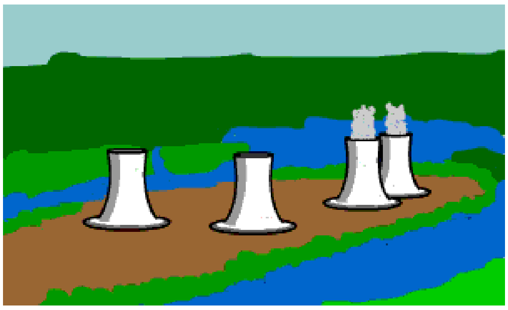


Also, when a plane (like The Condorde or the Space Shuttle) goes faster than the speed of sound, the result is a sonic boom. The change in air pressure (due to the extreme speed) creates a cone shaped wave that shoots out of the back of the plane. Just like the flashlight, guess what shape the cone hits the ground in?

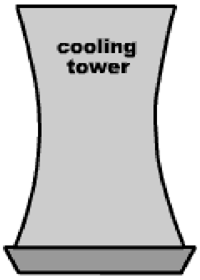


The sonic boom is heard (and felt) along the red edge of the hyperbola.

Hey, you've probably even seen a hyperbolic shaped building too.
Ever seen Three Mile Island or some other nuclear power plant?



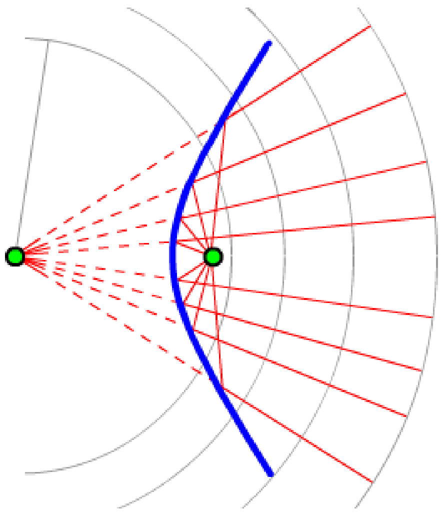
Those buildings are actually water cooling towers.



The hyperbolic shape helps the tower do its job more efficiently.

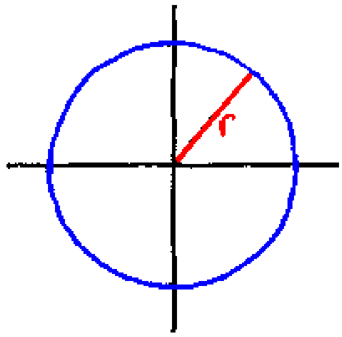
But, here's the really wild thing about the hyperbola -- Its reflective properties!

Check out the groovy geometry on this thing -- especially what is going on with that left focus:



Circles - The Formula and Graphing

Here's the formula for the circle:



$$x^2 + y^2 = r^2$$

(This is for when it's centered at the origin.)

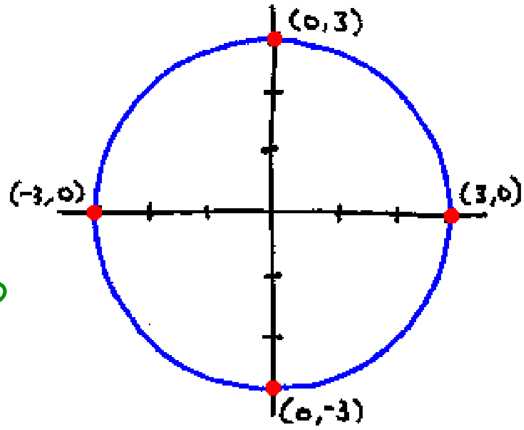
These are super easy to graph!

Let's graph this guy:

$$x^2 + y^2 = 9$$

So, the radius is 3.

r $\nearrow$ $3^2 = 9$ $\quad r = 3$



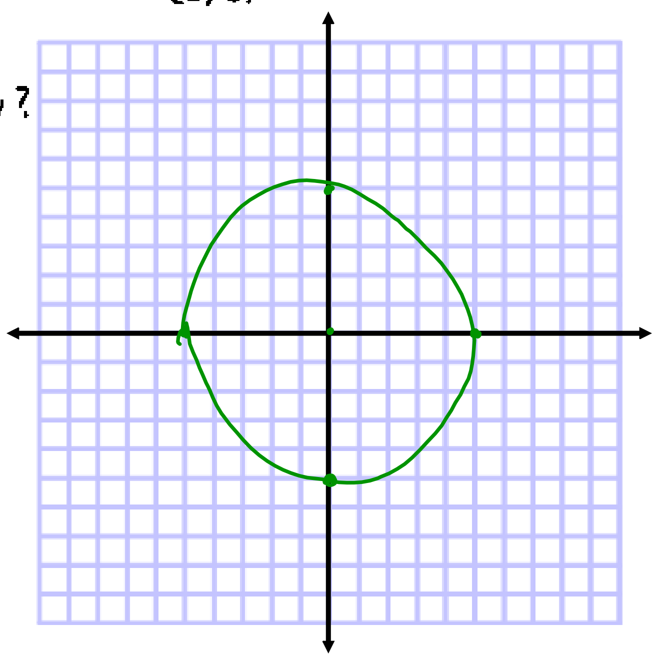
Didn't I tell you it was easy?

Your turn!

Graph $x^2 + y^2 = 25$

$x^2 + y^2 = r^2$

(0,0) $\quad r = 5$



Student Handout

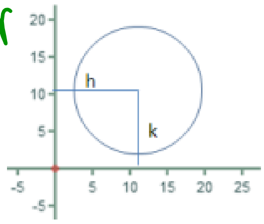
Recall from Basic Equation of a Circle, that when the circle's center is at the origin, the formula is

$$x^2 + y^2 = r^2$$

When the circle center is elsewhere, we need a more general form. We add two new variables h and k that are the coordinates of the circle center point:

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

center(h, k) radius r



We subtract these from x and y in the equation to translate ("move") the center back to the origin.

If you compare the two formulae, you will see that the only difference is that the h and k variables are subtracted from the x and y terms before squaring them:

Basic	$(x)^2 + (y)^2 = r^2$
General	$(x-h)^2 + (y-k)^2 = r^2$

Example

When we see the equation of a circle such as

$$(x-3)^2 + (y+2)^2 = 81$$

we know it is a circle of radius 9 with its center at x = 3, y = –2.

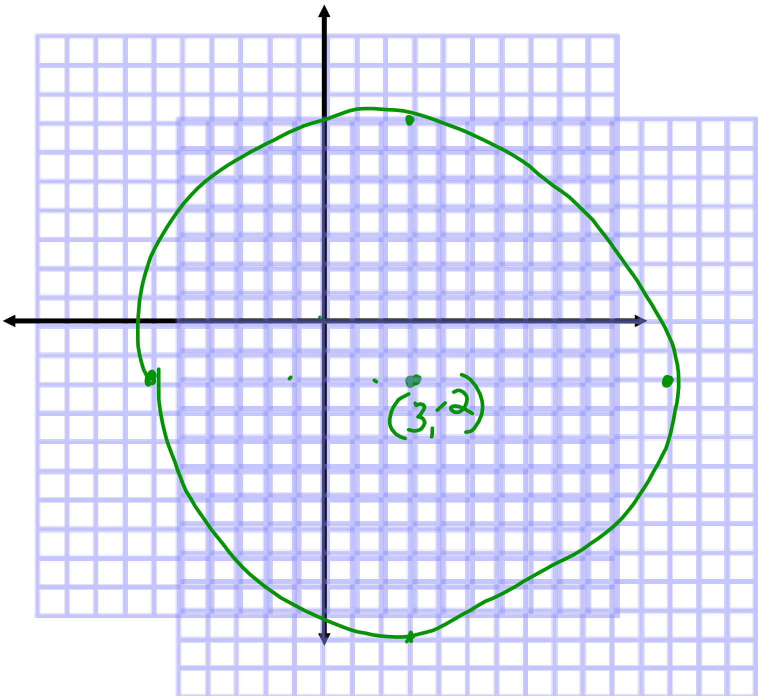
- The radius is 9 because the formula has r^2 on the right side. 9 squared is 81.
- The y coordinate is negative because the y term in the general equation is $(y-k)^2$.
  In the example, the equation has (y+2), so k must be negative: $(y-(-2))^2$ becomes $(y+2)^2$.

$(x-3)^2 + (y+2)^2 = 81$

$(x-h)^2 + (y-k)^2 = r^2$

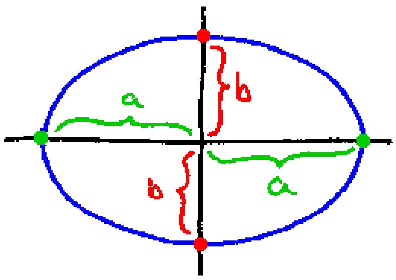
h=3 k=-2 r=9

center (3,-2) radius: 9



Ellipses - The Formula and Graphing

Here's the formula for the ellipse:



$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$$

Always a one!

(centered at the origin)

Look at some important stuff here:

$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$$

Notice that "a" is how wide we go to the right and left...

It make sense that it's under the x guy.

Notice that "b" is how high we go up and down...

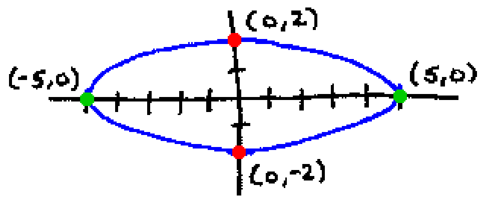
It makes sense that it's under the y guy.

Let's graph one:

$$\frac{x^2}{5^2}+\frac{y^2}{2^2}=1$$

↑ We go ←5→

↑ and ↑2↓



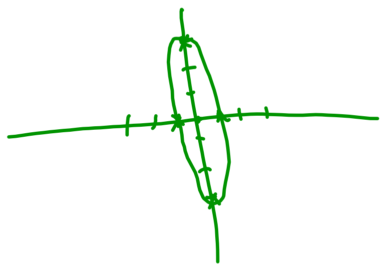
These are a bit tricky to draw from an artistic standpoint. Just try to not make them pointy!

Try it:

Graph $\boxed{\frac{x^2}{1^2}+\frac{y^2}{3^2}=1}$

left right

up down



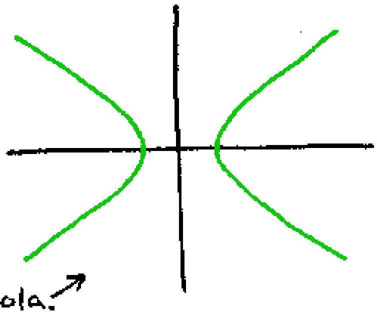
Hyperbolas - The Formula and Graphing

If you can graph an ellipse, you can graph a hyperbola! (You'll see.)

Here are the formulas for the hyperbola:

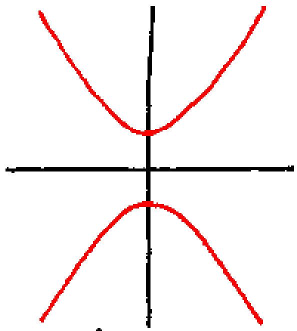
$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

If the x is in front, then it's a sideways hyperbola.



$$\boxed{\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1}$$

If the y is in front, then it's an up and down hyperbola.



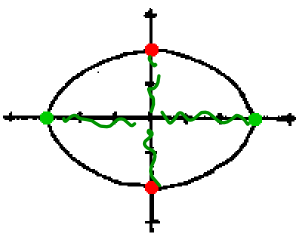
* Did you notice the minus signs? That's how these differ from ellipses!

Don't start freaking -- it's pretty easy!

Let's do one... and pretend it's an ellipse:

$$\frac{x^2}{3^2} - \frac{y^2}{2^2} = 1$$

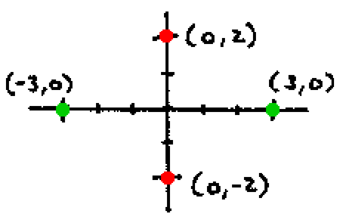
The ellipse would be:



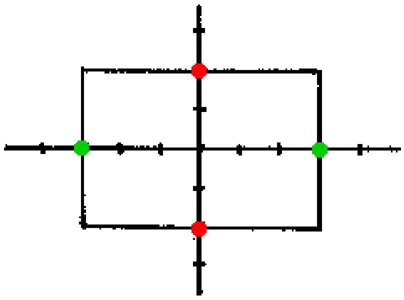
$$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$$

ellipse

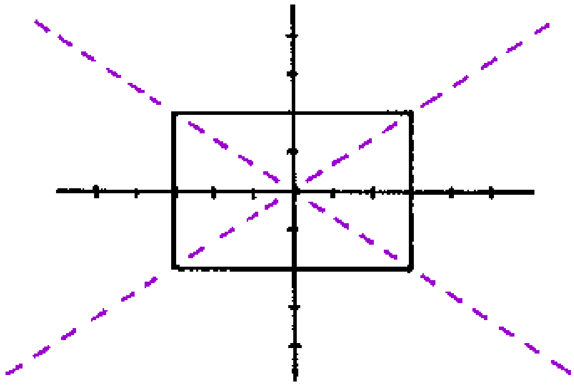
But, instead of completely drawing the ellipse, let's just plot the main points:



Draw a rectangle using these points:



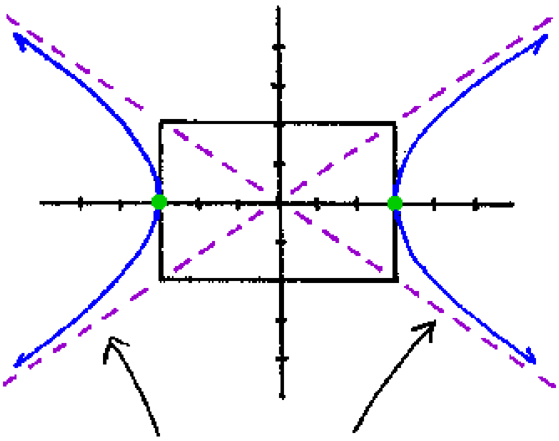
Draw lines through the corners:



Since x is in front, it's a sideways hyperbola:

left right

$$\frac{x^2}{3^2} - \frac{y^2}{2^2} = 1$$



These lines are asymptotes.
Graphs hug asymptotes -- they get closer and closer but never touch.

That wasn't too bad, was it?

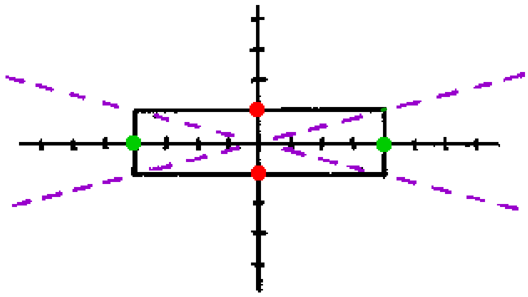
These are more like little art projects than math problems.

Let's do another one:

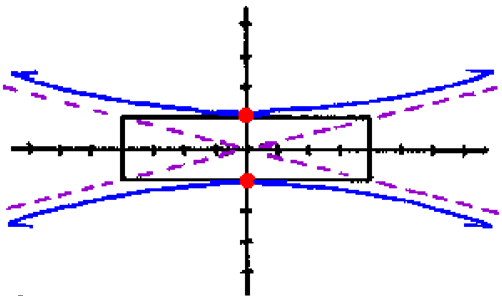
$$\frac{y^2}{1} - \frac{x^2}{16} = 1$$

Make the rectangle:

$$\frac{y^2}{1^2} - \frac{x^2}{4^2} = 1$$



y is in front, so it's an up and down hyperbola.

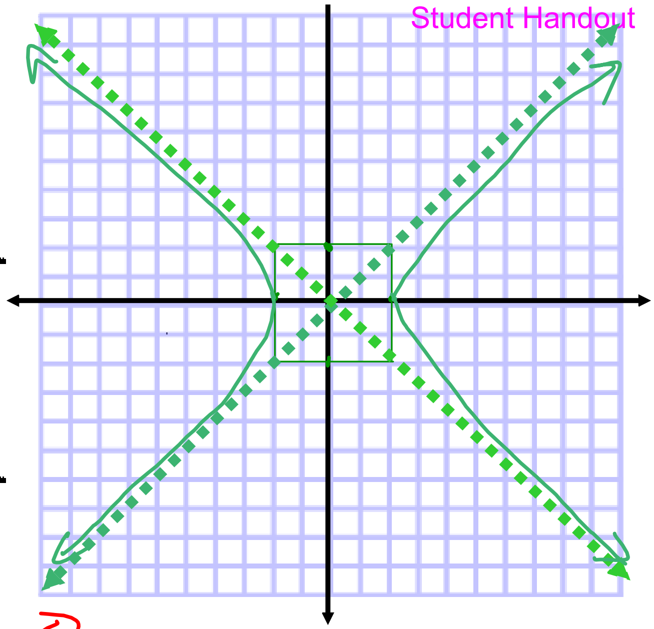


This one's really a flat wide guy!

Your turn:

Graph 1 $\boxed{\frac{x^2}{4} - \frac{y^2}{4} = 1}$

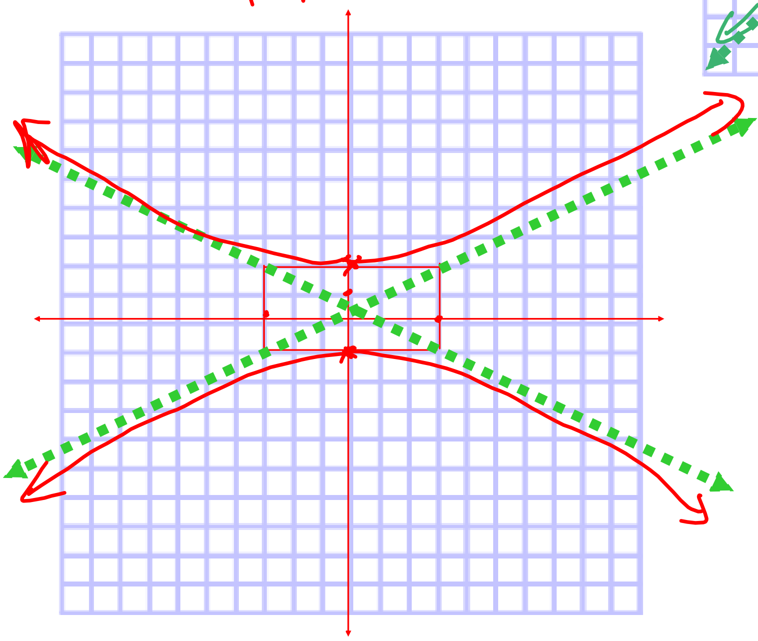
$\frac{x^2}{2^2} - \frac{y^2}{2^2} = 1$



Graph 2 $\boxed{\frac{y^2}{1} - \frac{x^2}{9} = 1}$

$\frac{y^2}{1^2} - \frac{x^2}{3^2} = 1$

up/down



Student Handout

Identify the center and radius of each. Then sketch the graph.

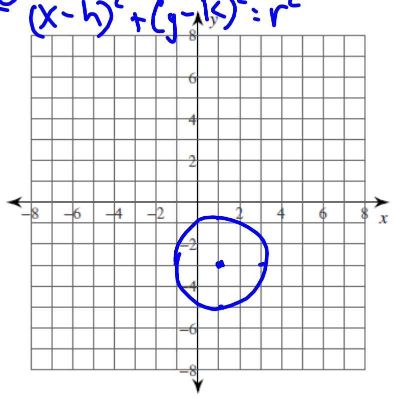
1) $(x-1)^2 + (y+3)^2 = 4$

$(x-h)^2 + (y-k)^2 = r^2$

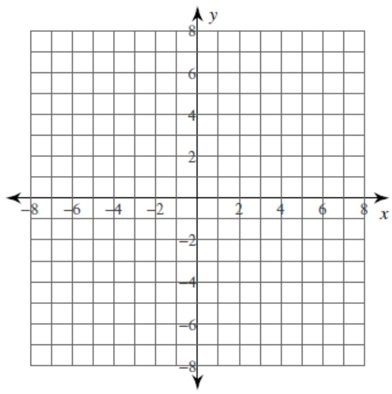
center $(1,-3)$

(h,k)

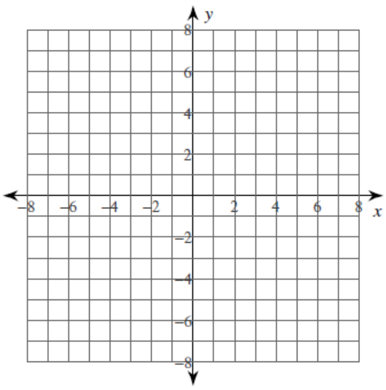
$r = 2$



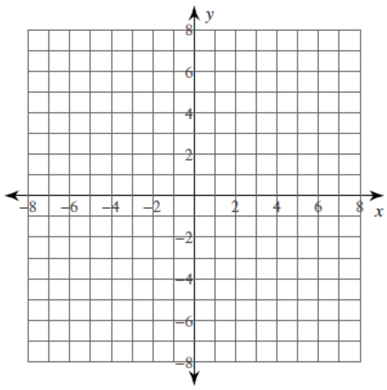
2) $(x-2)^2 + (y+1)^2 = 16$



3) $(x-1)^2 + (y+4)^2 = 9$

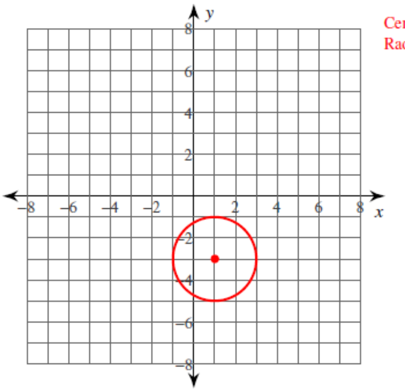


4) $x^2 + (y-3)^2 = 14$



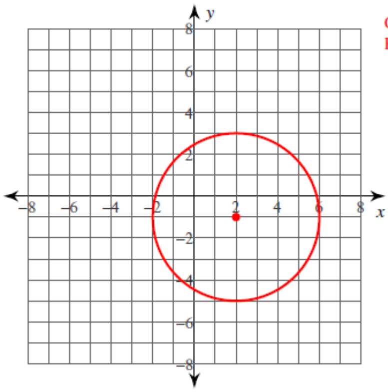
Identify the center and radius of each. Then sketch the graph.

1) $(x-1)^2+(y+3)^2=4$



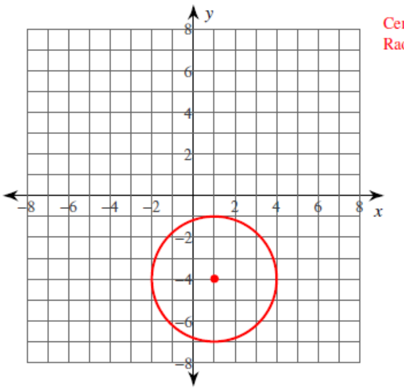
Center: (1, −3)
Radius: 2

2) $(x-2)^2+(y+1)^2=16$



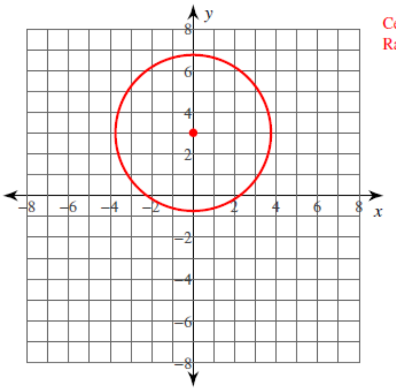
Center: (2, −1)
Radius: 4

3) $(x-1)^2+(y+4)^2=9$



Center: (1, −4)
Radius: 3

4) $x^2+(y-3)^2=14$



Center: (0, 3)
Radius: $\sqrt{14}$

Conic Sections

Student Handout

What you need to know:

Equation of a Circle. The equation of a circle with radius r centered at the origin $(0,0)$ is

$$x^2 + y^2 = r^2$$

The equation of a circle with radius r centered at the point (h,k) is

$$(x-h)^2 + (y-k)^2 = r^2$$

Equation of an Ellipse. The equation of an ellipse centered at the origin $(0,0)$ is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The equation of a centered at the point (h,k) is

$$\boxed{\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1}$$

Equation of a Hyperbola. The equation of a hyperbola "centered" at the origin $(0,0)$ with foci on the x-axis is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The asymptotes are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.

The equation of a hyperbola "centered" at the origin $(0,0)$ with foci on the y-axis is

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

The asymptotes are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.

The equation of a hyperbola "centered" at the point (h,k) is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

or

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Student Handout

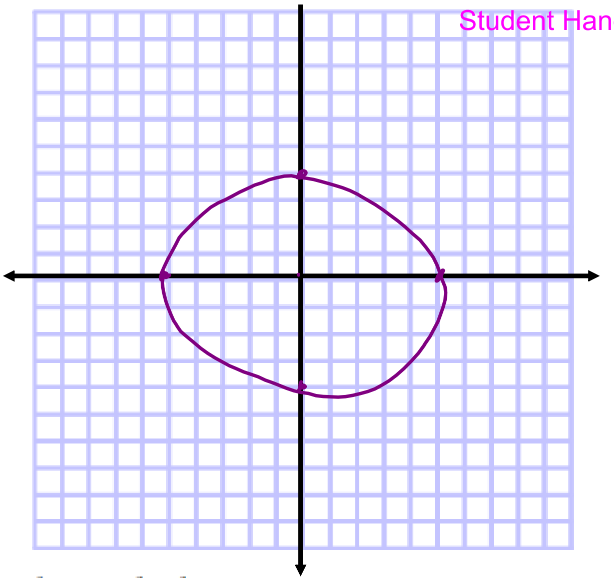
$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$a=5$ $b=4$

$\frac{x^2}{5^2}+\frac{y^2}{4^2}=1$



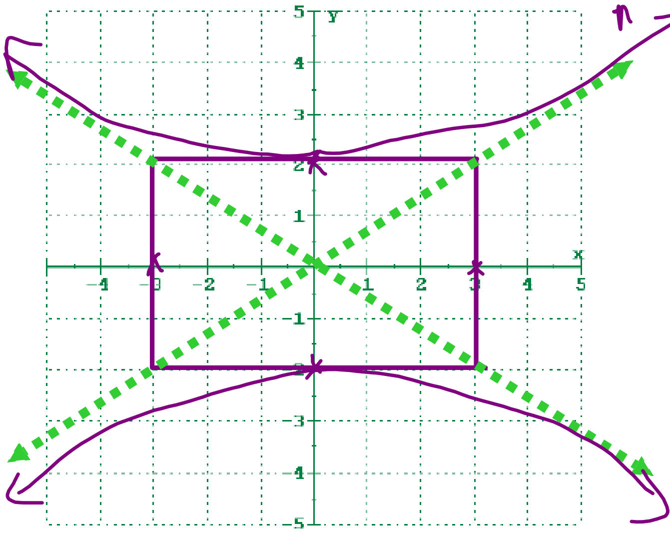
Practice: Sketch the following hyperbolas.

$$\frac{y^2}{4}-\frac{x^2}{9}=1$$

$\cup$ $\cap$

$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$

$b=2$ $a=3$

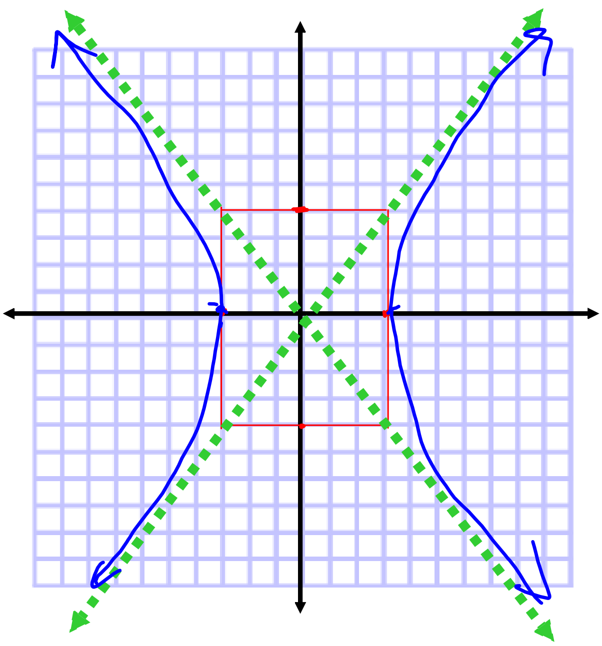


hyperbola

) (

$$\frac{x^2}{9}-\frac{y^2}{16}=1$$

$\frac{x^2}{3^2}-\frac{y^2}{4^2}=1$



Student Handout

Graph the given equation. $y=a(x-h)^2+k$ vertex (h,k)

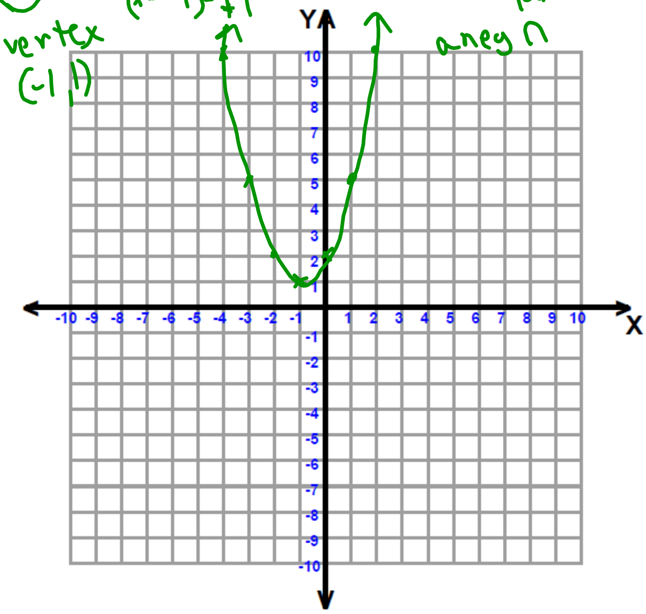
a pos $\cup$ $|a|>1$ $\cup$ $|a|<1$

a neg $\cap$

1) $y = (x + 1)^2 + 1$

$(x - -1)^2 + 1$

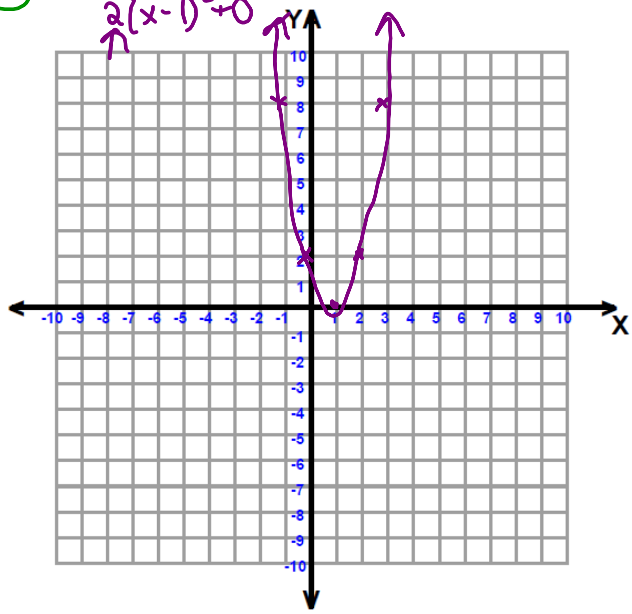
vertex $(-1,1)$



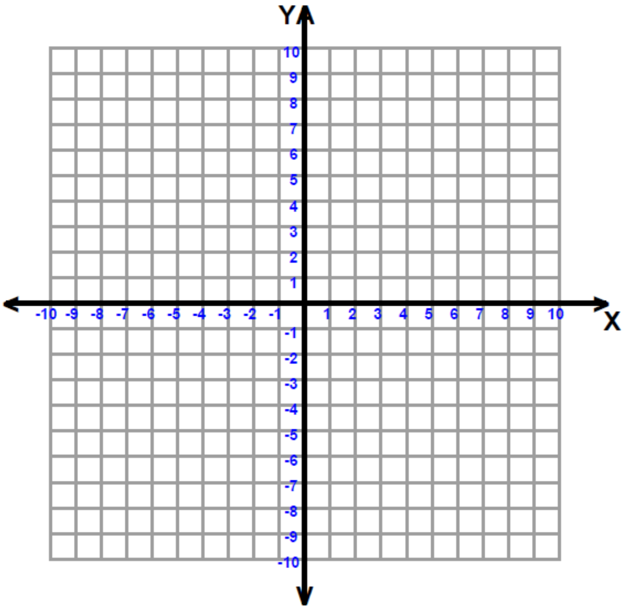
2) $y = 2(x - 1)^2$

vertex $(1,0)$

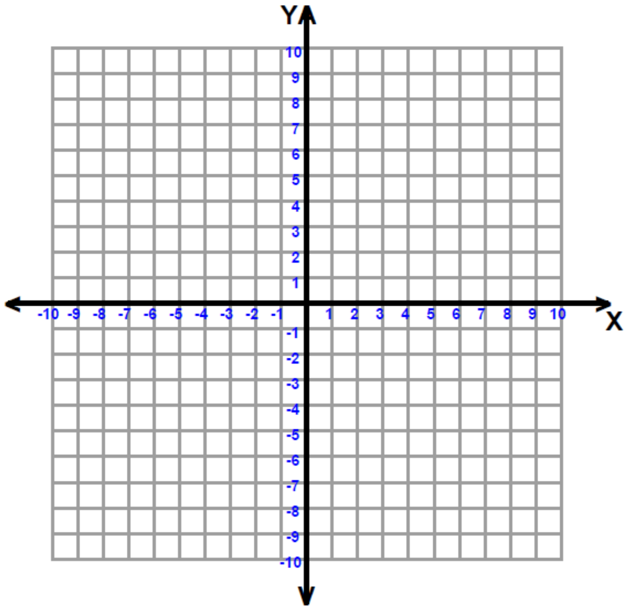
$2(x-1)^2+0$



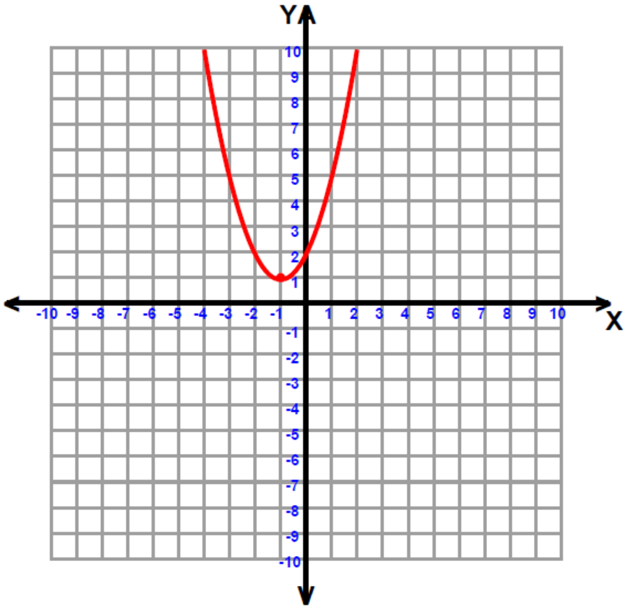
3) $y = (x + 2)^2$



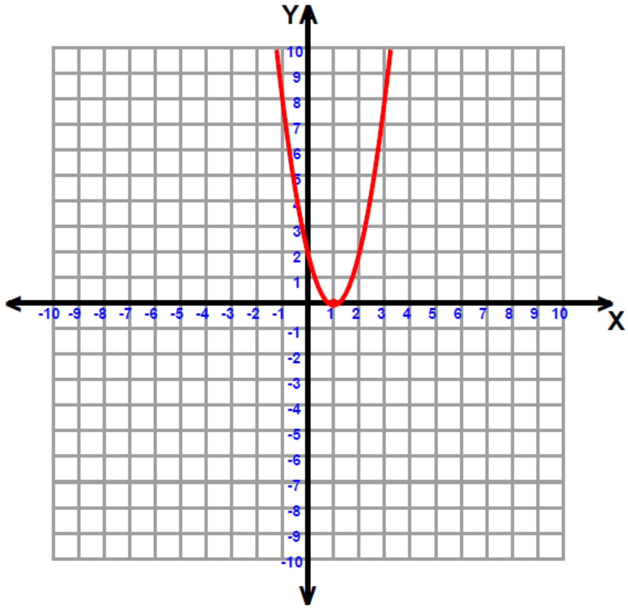
4) $y = -2(x - 1)^2$



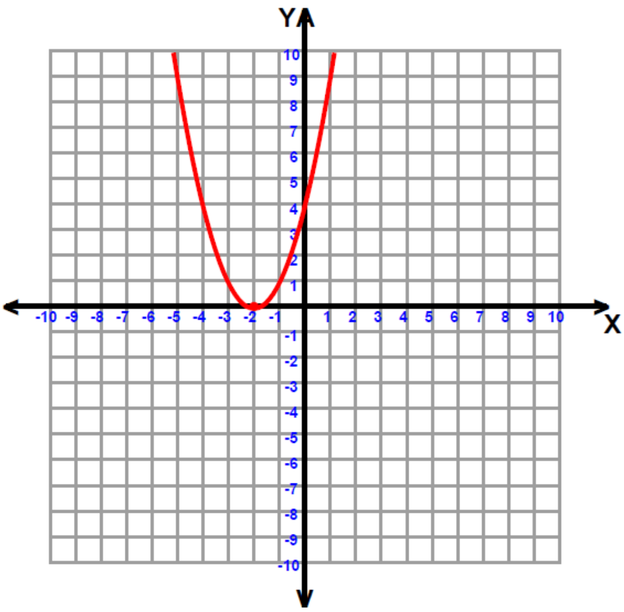
1) $y = (x + 1)^2 + 1$



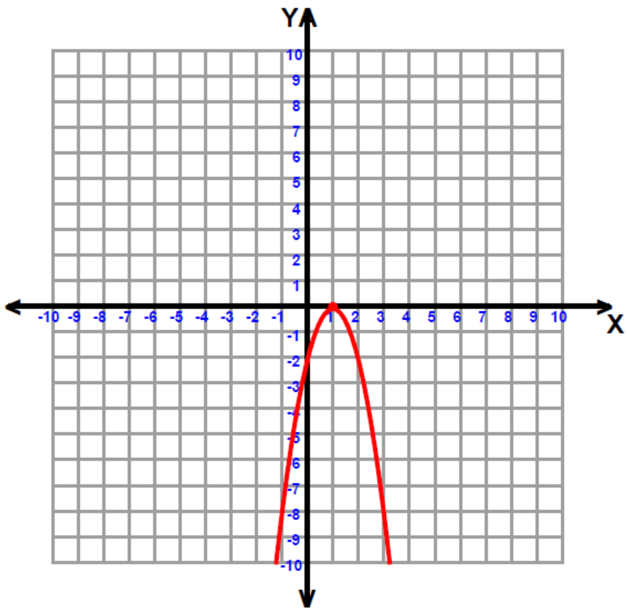
2) $y = 2(x - 1)^2$



3) $y = (x + 2)^2$



4) $y = -2(x - 1)^2$



Student Handout

4) $y = -2(x + 5)^2 + 18$

$x = -5$

Min/Max value: $(-5, 18)$

~~y-int:~~

~~x-int:~~

Vertex: $(-5, 18)$

Axis of Symmetry: $x = -5$

Opens: down

6) $y = 2(x + 1)^2 - 8$

Min/Max value:

~~y-int:~~

~~x-int:~~

Vertex:

Axis of Symmetry:

Opens:

4) $y = -2(x + 5)^2 + 18$

Max value: 18

x-int: -2 , -8

y-int: -32

Vertex = (-5 , 18)

Axis of Symmetry: x = -5

Opens: Down

6) $y = 2(x + 1)^2 - 8$

Min value: -8

x-int: -3 , 1

y-int: -6

Vertex = (-1 , -8)

Axis of Symmetry: x = -1

Opens: Up