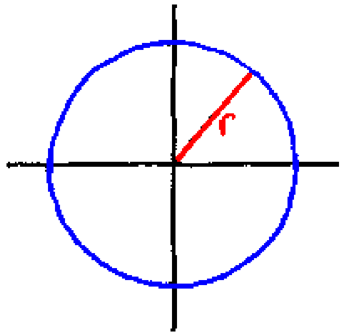
CONIC SECTIONS INTRO Precalc Student Handout BLANK

Circles - The Formula and Graphing

Here's the formula for the circle:



$$x^2 + y^2 = r^2$$

(This is for when it's centered at the origin.)

These are super easy to graph!

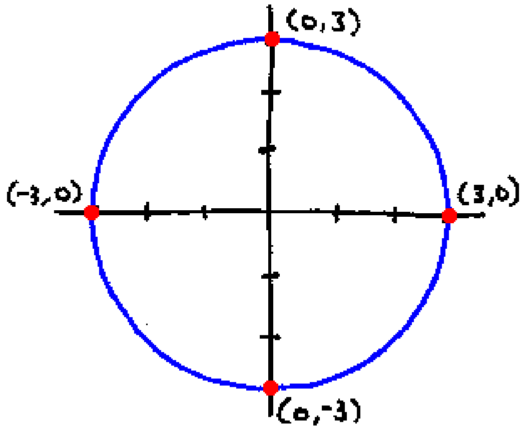
Let's graph this guy:

$$x^2 + y^2 = 9$$

So, the radius is 3.

$3^2 = 9$

r



Didn't I tell you it was easy?

Your turn!

Graph $x^2 + y^2 = 25$

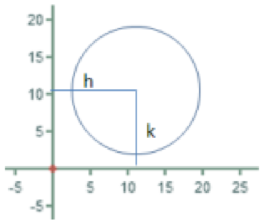
Recall from Basic Equation of a Circle, that when the circle's center is at the origin, the formula is

$$x^2 + y^2 = r^2$$

When the circle center is elsewhere, we need a more general form. We add two new variables h and k that are the coordinates of the circle center point:

$$(x-h)^2 + (y-k)^2 = r^2$$

We subtract these from x and y in the equation to translate ("move") the center back to the origin.



If you compare the two formulae, you will see that the only difference is that the h and k variables are subtracted from the x and y terms before squaring them:

Basic	$(x)^2 \quad + (y)^2 \quad = r^2$
General	$(x-h)^2 + (y-k)^2 = r^2$

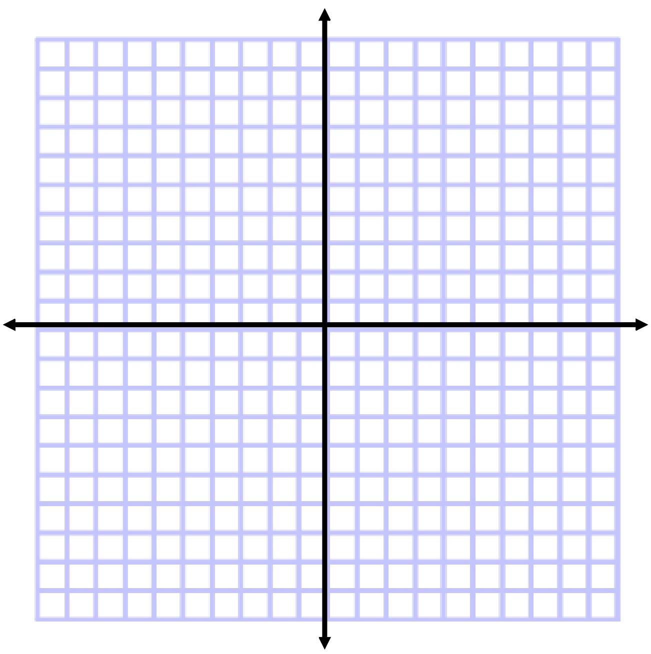
Example

When we see the equation of a circle such as

$$(x-3)^2 + (y+2)^2 = 81$$

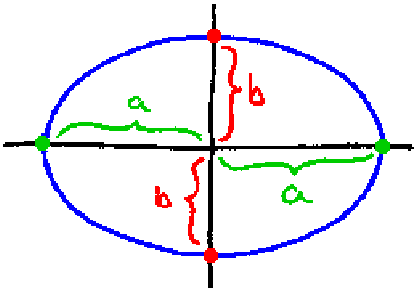
we know it is a circle of radius 9 with its center at $x = 3, \quad y = -2$.

- The radius is 9 because the formula has r^2 on the right side. 9 squared is 81.
- The y coordinate is negative because the y term in the general equation is $(y-k)^2$.
  In the example, the equation has (y+2), so k must be negative: $(y-(-2))^2$ becomes $(y+2)^2$.



Ellipses - The Formula and Graphing

Here's the formula for the ellipse:



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Always a one!

(centered at the origin)

Look at some important stuff here:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Notice that "a" is how wide we go to the right and left...

$\longleftrightarrow$

It make sense that it's under the x guy.

Notice that "b" is how high we go up and down...

$\updownarrow$

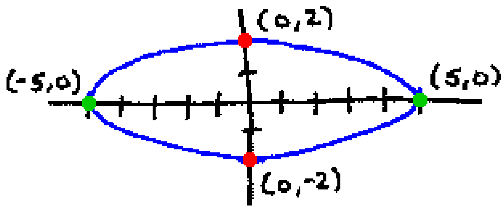
It makes sense that it's under the y guy.

Let's graph one:

$$\frac{x^2}{5^2} + \frac{y^2}{2^2} = 1$$

$\uparrow$ We go $\leftarrow 5 \rightarrow$

$\uparrow$ and $\uparrow$ 2 $\downarrow$



These are a bit tricky to draw from an artistic standpoint. Just try to not make them pointy!

Try it:

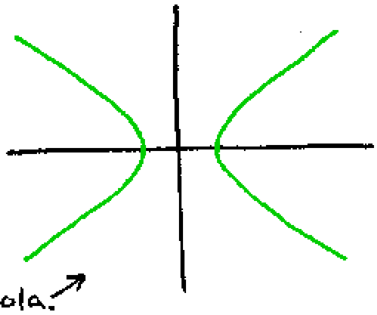
Graph $\frac{x^2}{1^2} + \frac{y^2}{3^2} = 1$

Hyperbolas - The Formula and Graphing

If you can graph an ellipse, you can graph a hyperbola! (You'll see.)

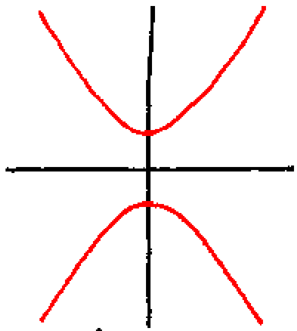
Here are the formulas for the hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



If the x is in front, then it's a sideways hyperbola.

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$



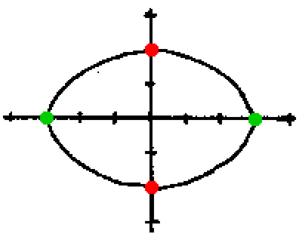
If the y is in front, then it's an up and down hyperbola.

* Did you notice the minus signs? That's how these differ from ellipses!

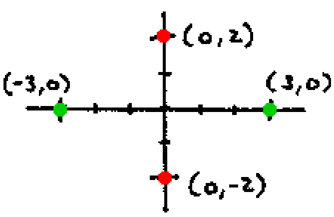
Don't start freaking -- it's pretty easy!
Let's do one ... and pretend it's an ellipse:

The ellipse would be:

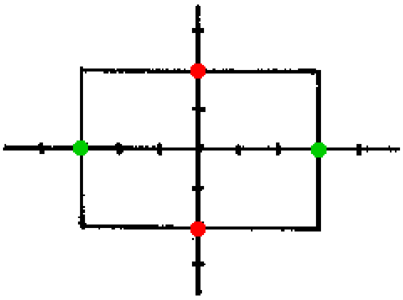
$$\frac{x^2}{3^2} - \frac{y^2}{2^2} = 1$$



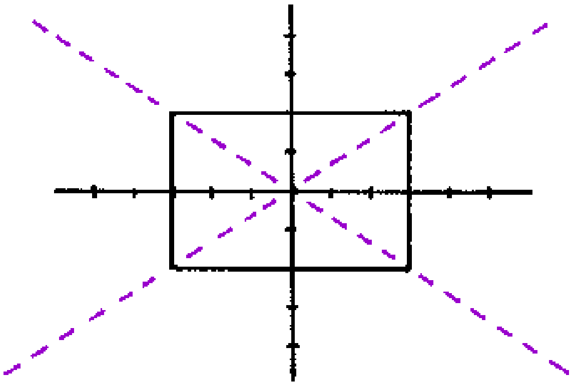
But, instead of completely drawing the ellipse, let's just plot the main points:



Draw a rectangle using these points:

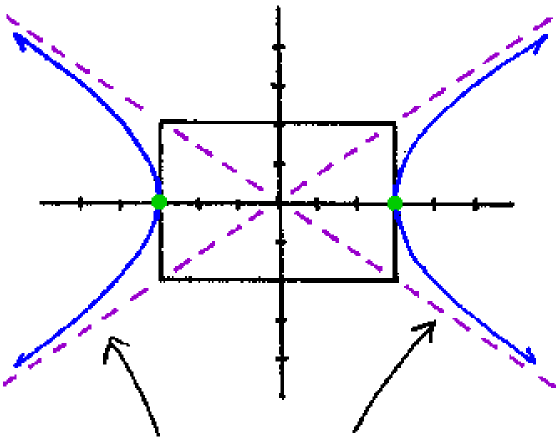


Draw lines through the corners:



Since x is in front, it's a sideways hyperbola:

$$\frac{x^2}{3^2} - \frac{y^2}{2^2} = 1$$



These lines are asymptotes.
Graphs hug asymptotes -- they get closer and closer but never touch.

That wasn't too bad, was it?

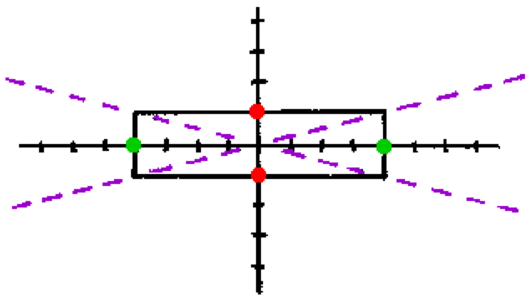
These are more like little art projects than math problems.

Let's do another one:

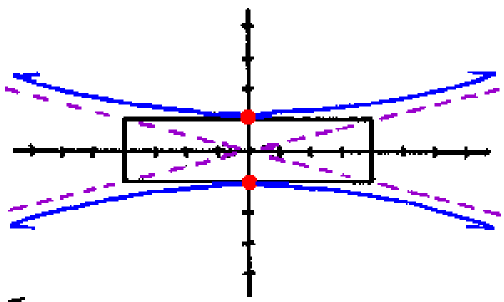
$$\frac{y^2}{1} - \frac{x^2}{16} = 1$$

Make the rectangle:

$$\frac{y^2}{1^2} - \frac{x^2}{4^2} = 1$$



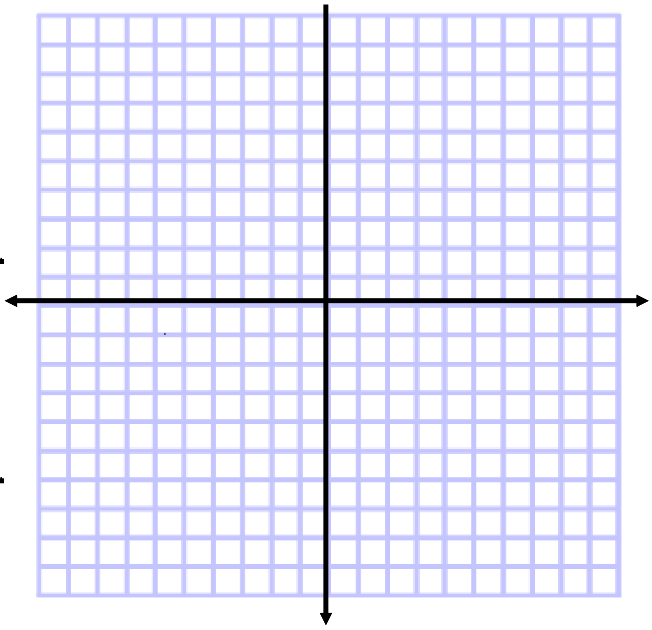
y is in front, so it's an up and down hyperbola.



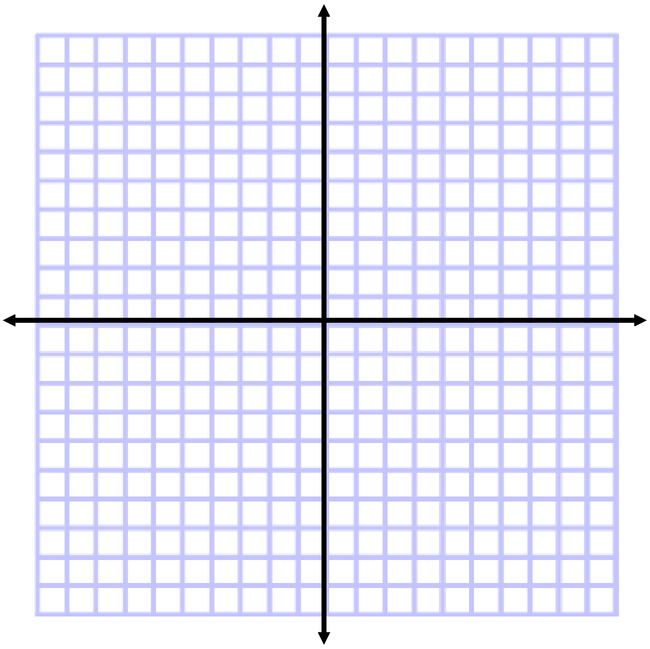
This one's really a flat wide guy!

Your turn:

Graph $\frac{x^2}{4} - \frac{y^2}{4} = 1$



Graph $\frac{y^2}{1} - \frac{x^2}{9} = 1$



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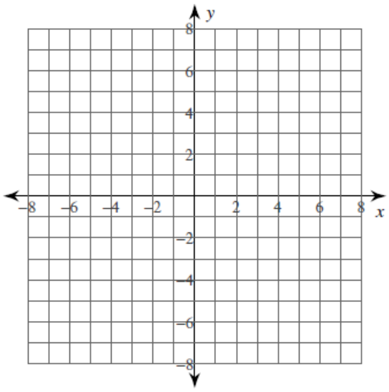
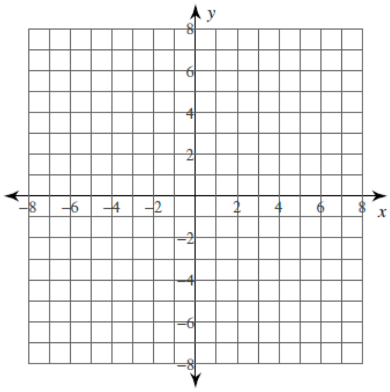
Identify the center and radius of each. Then sketch the graph.

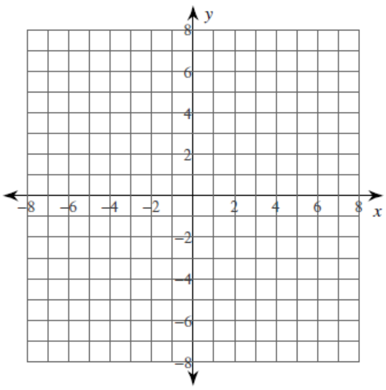
1) $(x-1)^2 + (y+3)^2 = 4$



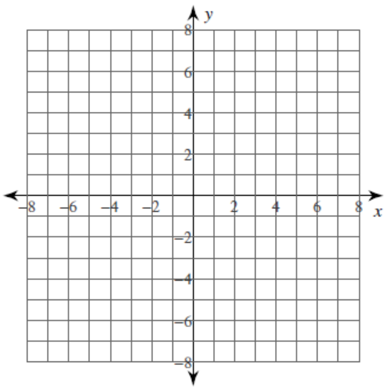
2) $(x-2)^2 + (y+1)^2 = 16$



3) $(x-1)^2 + (y+4)^2 = 9$



4) $x^2 + (y-3)^2 = 14$



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Conic Sections

What you need to know:

Equation of a Circle. The equation of a circle with radius r centered at the origin $(0, 0)$ is

$$x^2 + y^2 = r^2$$

The equation of a circle with radius r centered at the point (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2$$

Equation of an Ellipse. The equation of an ellipse centered at the origin $(0, 0)$ is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The equation of a centered at the point (h, k) is

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Equation of a Hyperbola. The equation of a hyperbola "centered" at the origin $(0, 0)$ with foci on the x-axis is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The asymptotes are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.

The equation of a hyperbola "centered" at the origin $(0, 0)$ with foci on the y-axis is

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

The asymptotes are $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$.

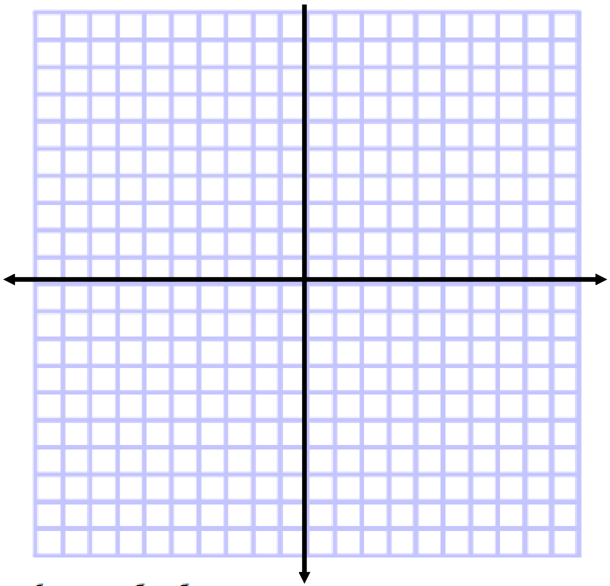
The equation of a hyperbola "centered" at the point (h, k) is

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

or

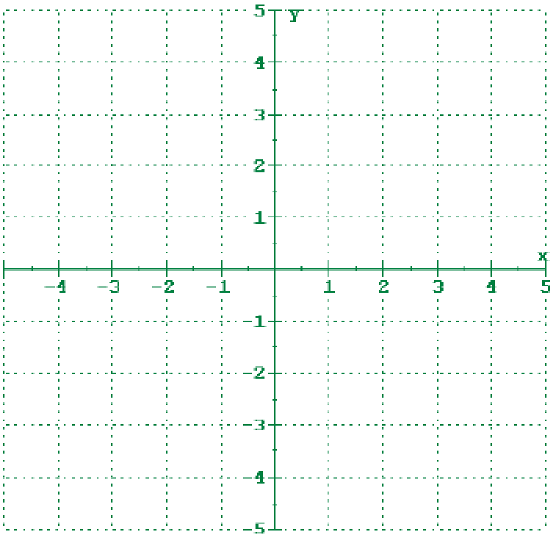
$$\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$$

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

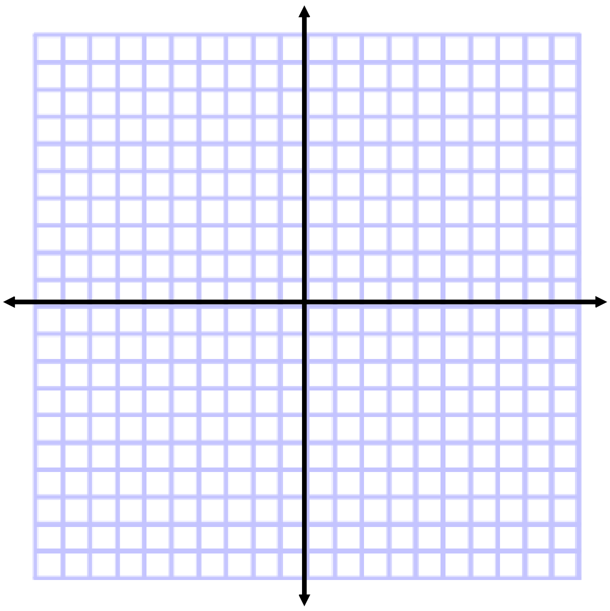


Practice: Sketch the following hyperbolas.

$$\frac{y^2}{4}-\frac{x^2}{9}=1$$



$$\frac{x^2}{9}-\frac{y^2}{16}=1$$



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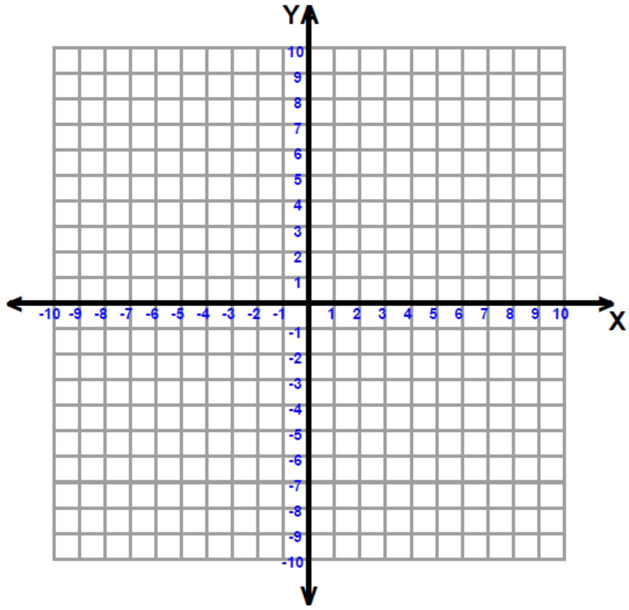
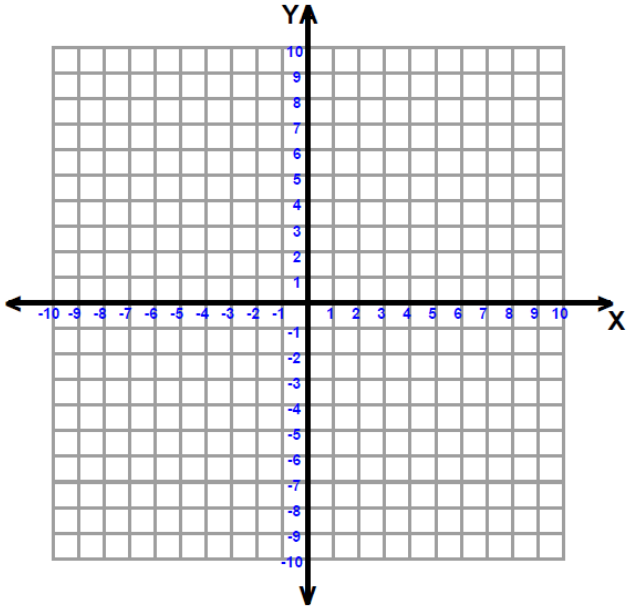
Graph the given equation.

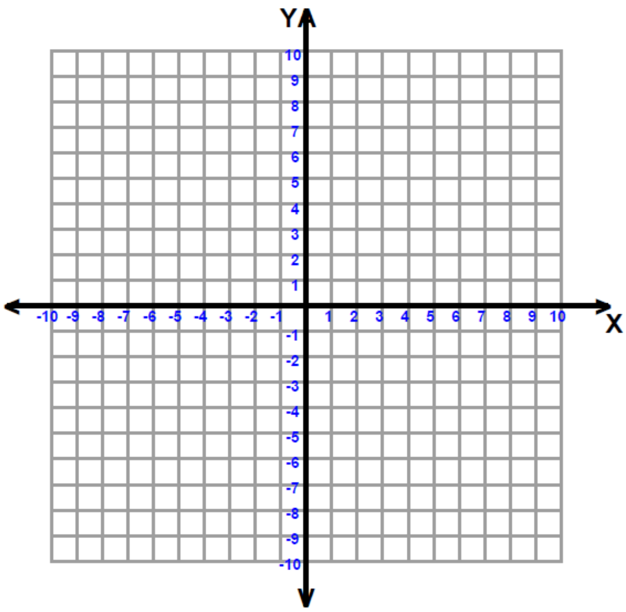
1) $y = (x + 1)^2 + 1$



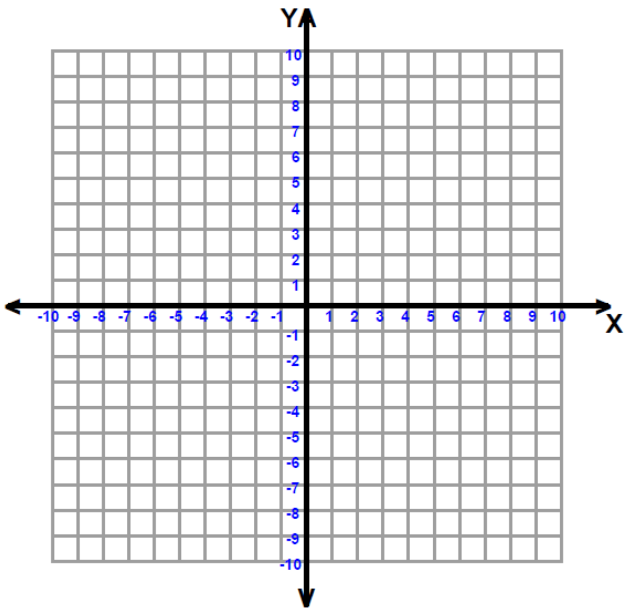
2) $y = 2(x - 1)^2$



3) $y = (x + 2)^2$



4) $y = -2(x - 1)^2$



4) $y = -2(x + 5)^2 + 18$

Min/Max value:

y-int:

x-int:

Vertex:

Axis of Symmetry:

Opens:

6) $y = 2(x + 1)^2 - 8$

Min/Max value:

y-int:

x-int:

Vertex:

Axis of Symmetry:

Opens: