TOC

9.3 Arcs and Chords

EVEN PAGE

EQ: Can you recognize and use relationships between arcs, chords, and diameters?

How are you doing? Write answer next to Essential Question
1. I don't understand the material
2. I understand a little.
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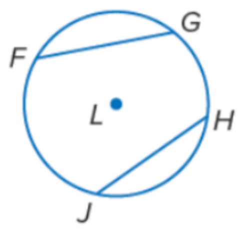
Summary: At least 3 sentences...

9.3 Examples Geo

Theorem 9.2

Words In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Example $\boxed{\overset{\frown}{FG} \cong \overset{\frown}{HJ} \text{ if and only if } \overline{FG} \cong \overline{HJ}.}$

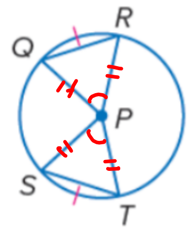


Proof Theorem 9.2 (part 1)

Given: $\odot P$; $\overset{\frown}{QR} \cong \overset{\frown}{ST}$

Prove: $\overline{QR} \cong \overline{ST}$

Proof:

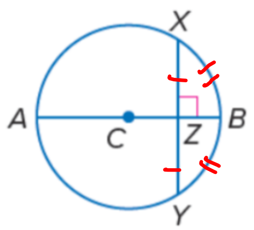


Statements	Reasons
1. $\odot P$, $\overset{\frown}{QR} \cong \overset{\frown}{ST}$	1. Given
2. $\angle QPR \cong \angle SPT$	2. If arcs are $\cong$, their corresponding central $\measuredangle$ are $\cong$.
3. $\overline{QP} \cong \overline{PR} \cong \overline{SP} \cong \overline{PT}$	3. All radii of a circle are $\cong$.
4. $\triangle PQR \cong \triangle PST$	4. SAS $\cong$
5. $\overline{QR} \cong \overline{ST}$	5. CPCTC

Theorems

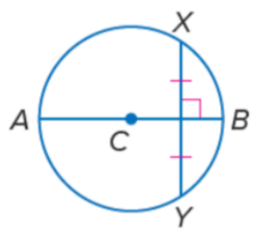
9.3 If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.

Example If diameter $\overline{AB}$ is perpendicular to chord $\overline{XY}$, then $\overline{XZ} \cong \overline{ZY}$ and $\overset{\frown}{XB} \cong \overset{\frown}{BY}$.



9.4 The perpendicular bisector of a chord is a diameter (or radius) of the circle.

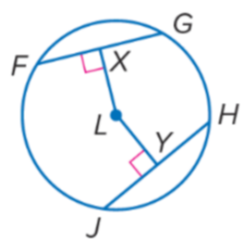
Example If $\overline{AB}$ is a perpendicular bisector of chord $\overline{XY}$, then $\overline{AB}$ is a diameter of $\odot C$.



Theorem 9.5

Words In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

Example $\overline{FG} \cong \overline{JH}$ if and only if $LX = LY$.



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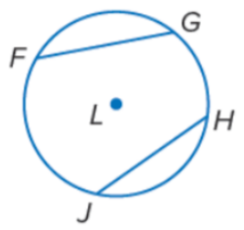

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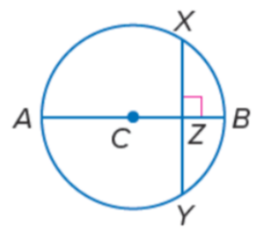

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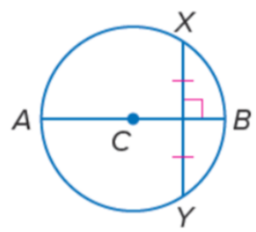

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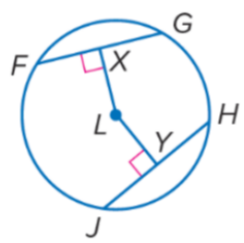
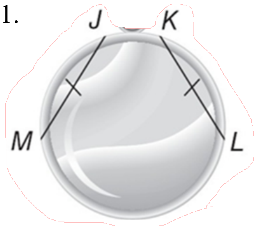


9.3 Examples Geo

1.



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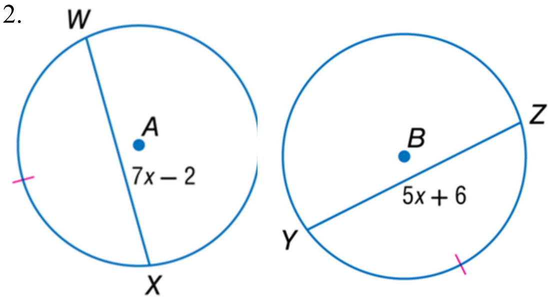

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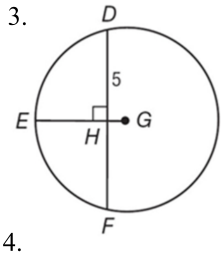

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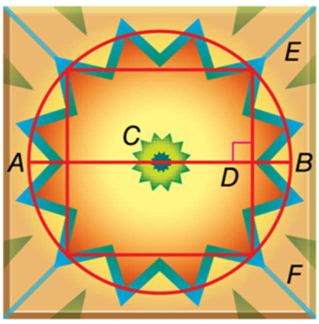

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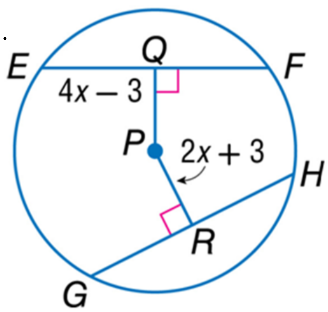


Real-World Example 1 **Use Congruent Chords to Find Arc Measure**

JEWELRY **A circular piece of jade is hung from a chain by two wires wrapped around the stone. $\overline{JM} \cong \overline{KL}$ and $m\overset{\frown}{KL} = 90$. Find $m\overset{\frown}{JM}$.**

Given: $\overline{JM} \cong \overline{KL}$

$m\overset{\frown}{KL} = 90$

Find: $m\overset{\frown}{JM}$

$\cong$ chords have $\cong$ arcs

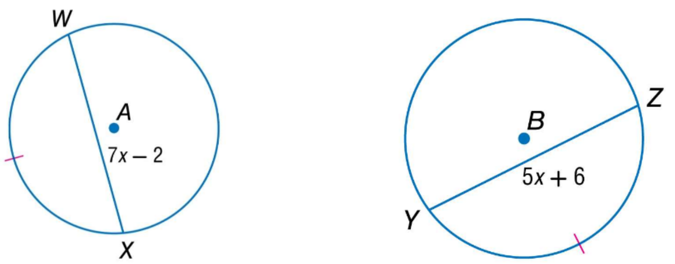
$m\overset{\frown}{JM} = m\overset{\frown}{KL}$

$m\overset{\frown}{JM} = 90$

9.3 Examples Geo

Example 2 Use Congruent Arcs to Find Chord Lengths

ALGEBRA In the figure, $\odot A \cong \odot B$ and $\overset{\frown}{WX} \cong \overset{\frown}{YZ}$. Find WX.



Given: $\odot A \cong \odot B$

$\overset{\frown}{WX} \cong \overset{\frown}{YZ}$

≅ Arcs have ≅ chords

Find: WX

$\overline{WX} \cong \overline{YZ}$

$WX = YZ$

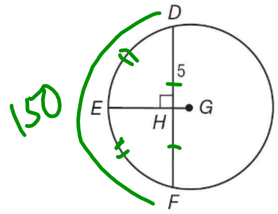
$$7x - 2 = 5x + 6$$
$$-5x \quad -5x$$
$$2x - 2 = 6$$
$$+2 \quad +2$$
$$\frac{2x}{2} = \frac{8}{2}$$
$$x = 4$$

$$WX = 7x - 2$$
$$= 7(4) - 2$$
$$= 28 - 2$$
$$WX = 26$$

9.3 Examples Geo

Example 3 Use a Radius Perpendicular to a Chord

In $\odot G$, $m\overparen{DEF} = 150$. Find $m\overparen{DE}$.



Given: $\odot G$, $m\overparen{DEF} = 150$

Find $m\overparen{DE}$

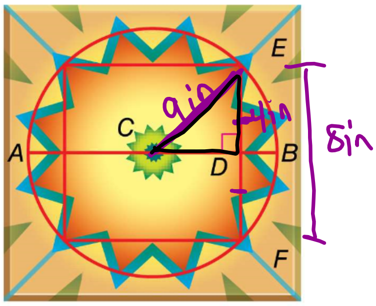
Radius $\overline{EG} \perp$ chord $\overline{DF}$

so $\overline{EG}$ bisects $\overparen{DEF}$

$m\overparen{DE} = \frac{150}{2} = 75$

Real-World Example 4 Use a Diameter Perpendicular to a Chord

CERAMIC TILE In the ceramic stepping stone below, diameter $\overline{AB}$ is 18 inches long and chord $\overline{EF}$ is 8 inches long. Find CD.



Given: Diameter is $\overline{AB}$
$AB = 18$ in
$EF = 8$ in

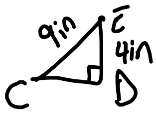
Find CD

Hint: Draw radius $\overline{CE}$ to make Right $\triangle CDE$

$CE = \frac{18}{2} = 9$ in

Also $\overline{AB} \perp \overline{EF}$ so $\overline{AB}$ bisects $\overline{EF}$

$ED = 4$ in



$CD^2 + 4^2 = 9^2$

$CD^2 = 9^2 - 4^2$

$CD = \sqrt{9^2 - 4^2} = \sqrt{81 - 16} = \sqrt{65}$

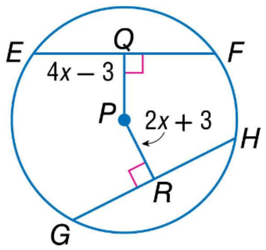
$CD = \sqrt{65}$

≈ 8.06 in

9.3 Examples Geo

Example 5 — **Chords Equidistant from Center**

ALGEBRA In $\odot P$, $EF = GH = 24$. Find PQ.



Given: $\odot P$, $EF = GH = 24$

Find: PQ

$\cong$ chords are equidistant from the center

$\overline{EF} \cong \overline{GH}$ so $\overline{PQ} \cong \overline{PR}$

$PQ = PR$

$4x - 3 = 2x + 3$

$-2x \quad -2x$

$2x - 3 = 3$

$+3 \quad +3$

$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

$PQ = 4x - 3$

$= 4 \cdot 3 - 3$

$= 12 - 3$

$PQ = 9$

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Write 3 Questions for this section on the left page

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Write this now.