

TOC ← **9.2 Measuring Angles and Arcs**

EQ: Can you use relationships between arcs and angles to find measures?

How are you doing? Write answer next to Essential Question

1. I don't understand the material
2. I understand a little.
3. I understand this material.
4. I could teach this to someone



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Summary: At least 3 sentences...

## 9.2 Examples Geo

odd page

### KEY TERMS

**Angles and Arcs** A **central angle** of a circle is an angle with a vertex in the center of the circle. Its sides contain two radii of the circle.  $\angle ABC$  is a central angle of  $\odot B$ .

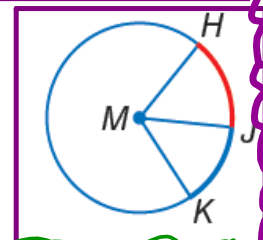
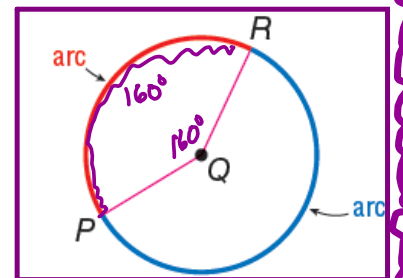
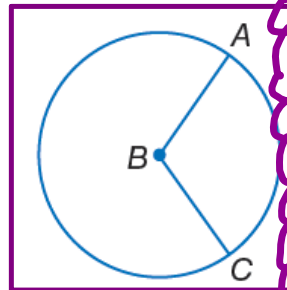
Recall from Lesson 1-4 that a *degree* is  $\frac{1}{360}$  of the circular rotation about a point.

*Measure an arc in degrees.  $m\widehat{RP} = 160^\circ$*

An **arc** is a portion of a circle defined by two endpoints. A central angle separates the circle into two arcs with measures related to the measure of the central angle.

**Congruent arcs** are arcs in the same or congruent circles that have the same measure.

**Adjacent arcs** are arcs in a circle that have exactly one point in common. In  $\odot M$ ,  $\widehat{HJ}$  and  $\widehat{JK}$  are adjacent arcs. As with adjacent angles, you can add the measures of adjacent arcs.



**Arc Length** **Arc length** is the distance between the endpoints along an arc measured in linear units. Since an arc is a portion of a circle, its length is a fraction of the circumference. *arc length is distance in ft, in, m, etc*

#### StudyTip

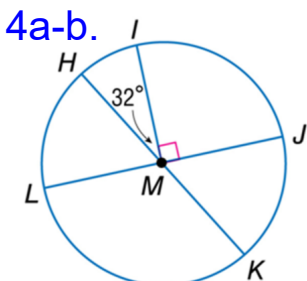
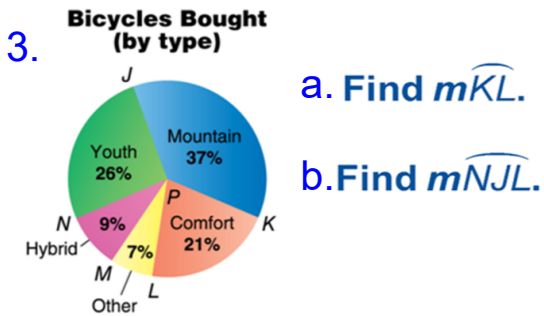
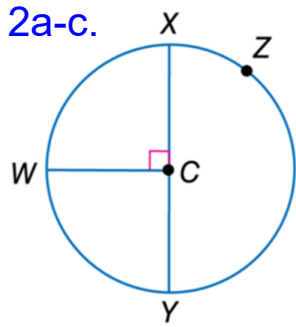
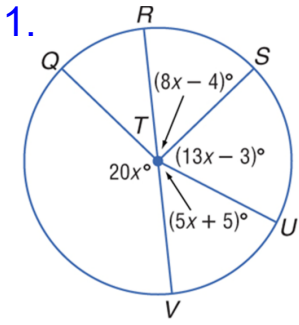
**Naming Arcs** Minor arcs are named by their endpoints. Major arcs and semicircles are named by their endpoints and another point on the arc that lies between these endpoints.

**Terminology** The terms **arc measure** and **arc length** are not interchangeable. Like angles, arcs have degree measure, denoted  $m\widehat{AC}$ . Just as with segment length **arc length** is a distance along a curve.

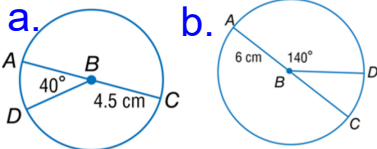
#### Watch Out!

**Arc Length** The length of an arc is given in linear units, such as centimeters. The measure of an arc is given in degrees.

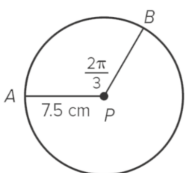
## 9.2 Examples Geo



5. Find the length of  $\widehat{DA}$ .



6. Find the length of  $\widehat{AB}$ .

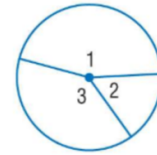


## 9.2 Examples Geo

### Key Concept Sum of Central Angles

**Words** The sum of the measures of the central angles of a circle with no interior points in common is 360.

**Example**  $m\angle 1 + m\angle 2 + m\angle 3 = 360$



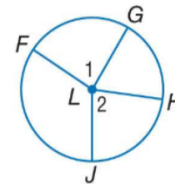
### Key Concept Arcs and Arc Measure

Arc	Measure
A <b>minor arc</b> is the shortest arc connecting two endpoints on a circle. <i>name with 2 or 3 pts</i>	The measure of a minor arc is less than 180 and equal to the measure of its related central angle. $m\widehat{AB} = m\angle ACB = x$
A <b>major arc</b> is the longest arc connecting two endpoints on a circle. <i>name with 3 pts</i>	The measure of a major arc is greater than 180, and equal to 360 minus the measure of the minor arc with the same endpoints. $m\widehat{ADB} = 360 - m\widehat{AB} = 360 - x$
A <b>semicircle</b> is an arc with endpoints that lie on a diameter. <i>name with 3 pts</i>	The measure of a semicircle is 180. $m\widehat{ADB} = 180$

### Theorem 10.1

**Words** In the same circle or in congruent circles, two minor arcs are congruent if and only if their central angles are congruent.

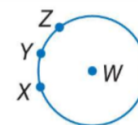
**Example** If  $\angle 1 \cong \angle 2$ , then  $\widehat{FG} \cong \widehat{HJ}$ .  
If  $\widehat{FG} \cong \widehat{HJ}$ , then  $\angle 1 \cong \angle 2$ .



### Postulate 10.1 Arc Addition Postulate

**Words** The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

**Example**  $m\widehat{XYZ} = m\widehat{XY} + m\widehat{YZ}$

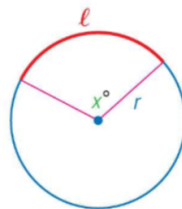


### Key Concept Arc Length

**Words** The ratio of the length of an arc  $\ell$  to the circumference of the circle is equal to the ratio of the degree measure of the arc to 360.

**Proportion**  $\frac{\ell}{2\pi r} = \frac{x}{360}$  or

**Equation**  $\ell = \frac{x}{360} \cdot 2\pi r$



$\frac{\ell}{2\pi r} = \frac{x}{360}$

$\ell = \frac{x}{360} \cdot 2\pi r$

$\theta = \frac{\ell}{r}$

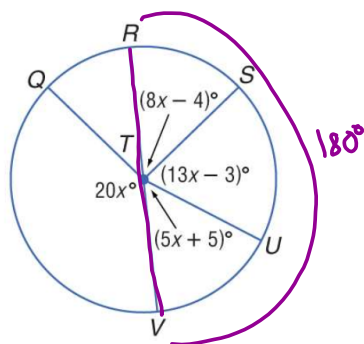
Arc length equation, when angle  $\theta$  is given in radians.

$\theta = \frac{\ell}{r}$

## 9.2 Examples Geo

### Example 1 Find Measures of Central Angles

Find the value of  $x$ .



①  $\overline{TV}$  is diameter

$$8x - 4 + 13x - 3 + 5x + 5 = 180$$

$$\begin{array}{r} 26x - 2 = 180 \\ +2 \quad +2 \\ \hline \end{array}$$

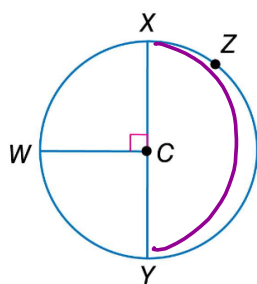
$$\begin{array}{r} 26x = 182 \\ \underline{26} \quad \underline{26} \\ \end{array}$$

$$x = 7$$

## Example 2

## Classify Arcs and Find Arc Measures

A.  $\overline{WC}$  is a radius of  $\odot C$ . Identify  $\widehat{XZY}$  as a *major arc*, *minor arc*, or *semicircle*. Then find its measure.



$\odot C$ , radius  $\overline{WC}$   
 a.  $\widehat{XZY}$   
 Semicircle  
 $m \widehat{XZY} = 180^\circ$

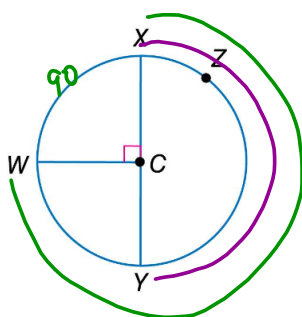
## 9.2 Examples Geo

### Example 2

### Classify Arcs and Find Arc Measures

B.  $\overline{WC}$  is a radius of  $\odot C$ . Identify  $\widehat{WZX}$  as a *major arc*, *minor arc*, or *semicircle*. Then find its measure.

$\odot C$ , radius  $\overline{WC}$



b.  $\widehat{WZX}$   
major arc  
 $m \widehat{WZX} = 360 - 90$   
 $= 270^\circ$

## 9.2 Examples Geo

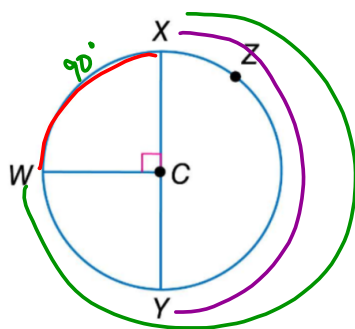
### Example 2

### Classify Arcs and Find Arc Measures

C.  $\overline{WC}$  is a radius of  $\odot C$ . Identify  $\widehat{XW}$  as a *major arc*, *minor arc*, or *semicircle*. Then find its measure.

$\odot C$ , radius  $\overline{WC}$

$\widehat{XW}$  minor arc  
 $m\widehat{XW} = 90^\circ$

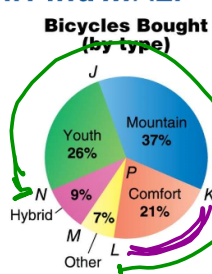




## Real-World Example 3

## Find Arc Measures in Circle Graphs

A. BICYCLES Refer to the circle graph. Find  $m\widehat{K\bar{L}}$ .



$$\begin{aligned} \textcircled{A} \quad m\widehat{K\bar{L}} &= 21\% \text{ of } 360 \\ &= .21 \cdot 360 \\ &= 75.6^\circ \end{aligned}$$

$$\begin{aligned} \textcircled{B} \quad m\widehat{N\bar{J}L} &= 84\% \text{ of } 360 \\ &= .84 \cdot 360 \\ &= 148^\circ \end{aligned}$$

Find %

$$\begin{array}{r} 26 \text{ or } 100\% \\ +37 \quad -16 \\ +21 \\ \hline 84\% \quad 84\% \end{array}$$

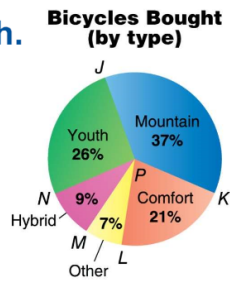
## 9.2 Examples Geo

### Real-World Example 3

### Find Arc Measures in Circle Graphs

**B. BICYCLES** Refer to the circle graph.

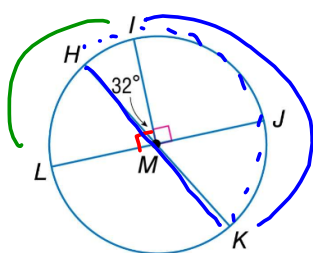
Find  $m\widehat{NJL}$ .



**Example 4**

**Use Arc Addition to Find Measures of Arcs**

A. Find  $m\widehat{LHI}$  in  $\odot M$ .



$\odot M$

(A)  $m\widehat{LHI} = 90^\circ$

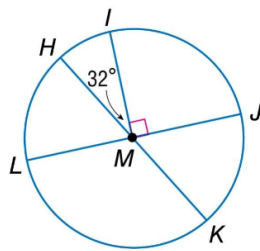
(B)  $m\widehat{IJK} = 180 - 32$   
 $= 148^\circ$

## 9.2 Examples Geo

### Example 4

Use Arc Addition to Find Measures of Arcs

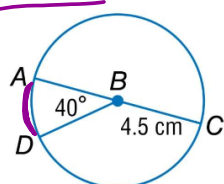
B. Find  $m\widehat{JK}$  in  $\odot M$ .



## Example 5

## Find Arc Length

A. Find the length of  $\widehat{DA}$ . Round to the nearest hundredth.



$$x = 40^\circ$$

$$r = 4.5 \text{ cm}$$

$$l = \frac{x}{360} \cdot 2\pi r$$

length of  $\widehat{DA}$

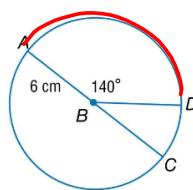
$$l = \frac{40}{360} \cdot 2\pi \cdot 4.5$$

$$l \approx 3.14 \text{ cm}$$

## Example 5

B. Find the length of  $\widehat{DA}$ . Round to the nearest hundredth.

Find Arc Length



$$m \widehat{DA} = 140^\circ$$

$$r = 6 \text{ cm}$$

$$l = \frac{x}{360} \cdot 2\pi r$$

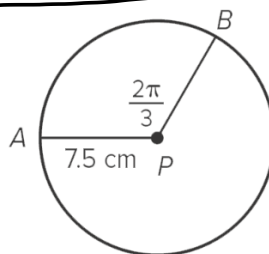
$$l = \frac{140}{360} \cdot 2\pi \cdot 6$$

$$l \approx \underline{14.66 \text{ cm}}$$

**Example 6****Find Arc Length Using Radian Measure**

Find the length of  $\widehat{AB}$ . Round to the nearest hundredth.

$$\begin{aligned} l &= r\theta \\ &= 7.5 \left( \frac{2\pi}{3} \right) \\ &\approx \underline{15.71 \text{ cm}} \end{aligned}$$



$$\theta = \frac{2\pi}{3}$$

$$r = 7.5 \text{ cm}$$

$$r\theta = \frac{l}{r} \cdot r$$

$$r\theta = l$$

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9.2 Measuring Angles and Arcs

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EQ: Can you use relationships between arcs and angles to find measures?

Write 3 Questions for this section on the left page

1. How are you doing?

Write answer next to the Summary

- 1: I don't understand the material.
- 2: I understand a little.
- 3: I understand this material.
- 4: I could teach this to someone.

Summary: At least 3 sentences...

Write this now.