4-7 The Law of Sines & the Law of Cosines

EVEN PAGE

TOC

EQ: Can you solve oblique triangles using the Law of Sines and the Law of Cosines and find areas of oblique triangles?

How are you doing? Write answer next to Essential Question
1. I don't understand the material
2. I understand a little.
3. I understand this material.
4. I could teach this to someone

Summary: At least 3 sentences...

New Vocabulary

- oblique triangles
- Law of Sines
- ambiguous case
- Law of Cosines
- Heron's Formula

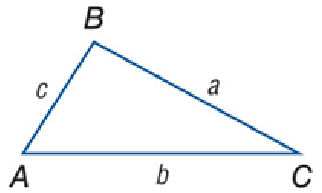
1 Solve Oblique Triangles

In Lesson 4-1, you used trigonometric functions to solve *right* triangles. In this lesson, you will solve **oblique triangles**—triangles that are not right triangles.

You can apply the **Law of Sines** to solve an oblique triangle if you know the measures of two angles and a nonincluded side (AAS), two angles and the included side (ASA), or two sides and a nonincluded angle (SSA).

Key Concept — Law of Sines

If $\triangle ABC$ has side lengths a, b, and c representing the lengths of the sides opposite the angles with measures A, B, and C, then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



StudyTip

Alternative Representations The Law of Sines can also be written in reciprocal form as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

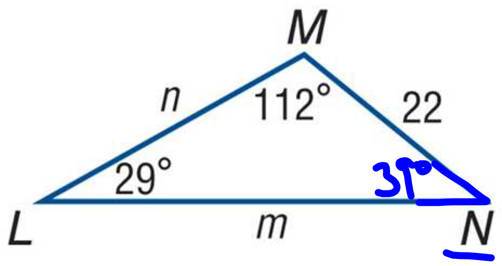
Must know a side & ∠ opposite each other

EXAMPLE 1 Apply the Law of Sines (AAS)

Solve ΔLMN. Round side lengths to the nearest tenth and angle measures to the nearest degree.

$N: 180 - (112 + 29)$

$N = 39^\circ$



$$\frac{\sin 29^\circ}{22} = \frac{\sin 112^\circ}{m}$$

$$m = \frac{22 \sin 112^\circ}{\sin 29^\circ}$$

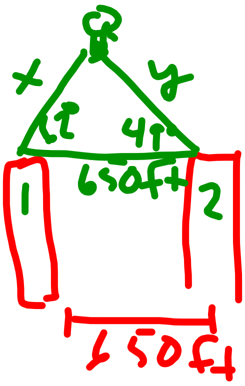
$m \approx 42.1$

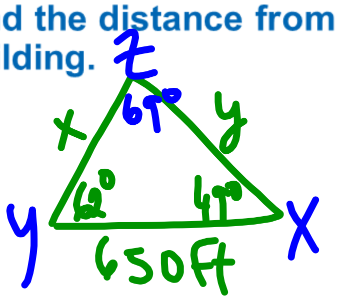
$$\frac{\sin 29^\circ}{22} = \frac{\sin 39^\circ}{n}$$

$$n = \frac{22 \sin 39^\circ}{\sin 29^\circ} \quad n \approx 28.6$$

Real-World EXAMPLE 2 Apply the Law of Sines (ASA)

BALLOONING The angle of elevation from the top of a building to a hot air balloon is 62º. The angle of elevation to the hot air balloon from the top of a second building that is 650 feet due east is 49º. Find the distance from the hot air balloon to each building.





$Z = 180 - (62 + 49)$

$Z = 69°$

$$\frac{\sin 69°}{650} = \frac{\sin 49°}{x}$$

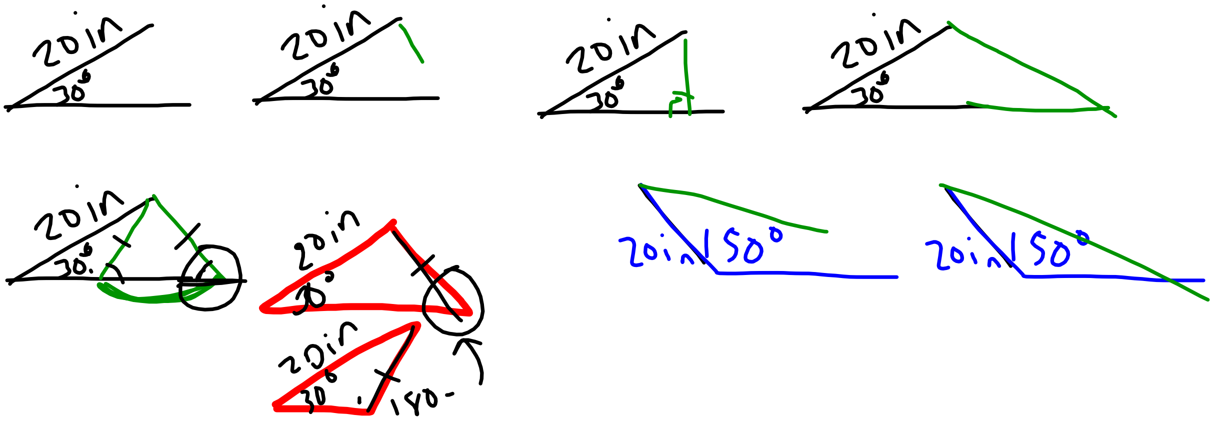
$$x = \frac{650 \sin 49°}{\sin 69°}$$

$x \approx 525.5\text{ft}$

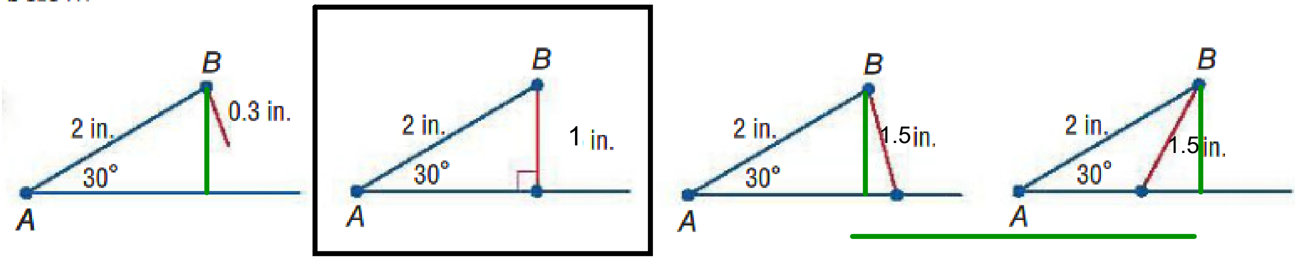
$$\frac{\sin 69°}{650} = \frac{\sin 62°}{y}$$

$$y = \frac{650 \sin 62°}{\sin 69°}$$

$y \approx 614.7\text{ft}$


20in
30°
20in
150°

From geometry, you know that the measures of two sides and a nonincluded angle (SSA) do not necessarily define a unique triangle. Consider the angle and side measures given in the figures below.

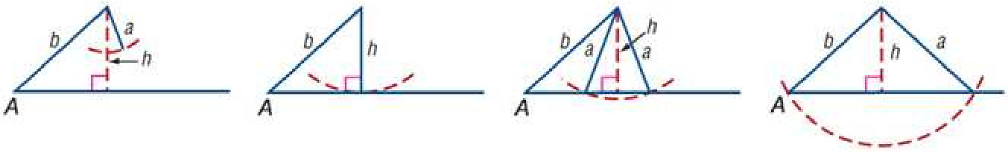


In general, given the measures of two sides and a nonincluded angle, one of the following will be true: (1) no triangle exists, (2) exactly one triangle exists, or (3) two triangles exist. In other words, when solving an oblique triangle for this **ambiguous case**, there may be no solution, one solution, or two solutions.

Key Concept **The Ambiguous Case (SSA)**

Consider a triangle in which a, b, and A are given. For the acute case $\boxed{\sin A = \frac{h}{b}}$, so $\boxed{h = b \sin A}$.

A is Acute. ($A < 90°$)



$a < b$ and $a < h$	$a < b$ and $a = h$	$a < b$ and $a > h$	$a \geq b$
no solution	one solution	two solutions	one solution

A is Right or Obtuse. ($A \geq 90°$)

$a \leq b$, no solution

$a > b$, one solution

To solve an ambiguous case oblique triangle, first determine the number of possible solutions. If the triangle has one or two solutions, use the Law of Sines to find them.

TechnologyTip

Using $\sin^{-1}$ Notice that when calculating $\sin^{-1}$ of a ratio, your calculator will never return two possible angle measures because $\sin^{-1}$ is a *function*. Also, your calculator will never return an obtuse angle measure for $\sin^{-1}$ because the inverse sine function has a range of $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ or $-90°$ to $90°$.

StudyTip

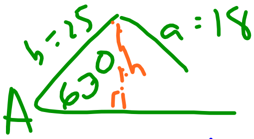
Make a Reasonable Sketch When solving triangles, a reasonably accurate sketch can help you determine whether your answer is feasible. In your sketch, check to see that the longest side is opposite the largest angle and that the shortest side is opposite the smallest angle.

EXAMPLE 3 The Ambiguous Case—One or No Solution

A. Find all solutions for the given triangle, if possible. If no solution exists, write no solution. Round side lengths to the nearest tenth and angle measures to the nearest degree.
$A = 63°$, $a = 18$, $b = 25$

acute ∠A
a < b
18 < 25

3a



h = 25 sin 63
h ≈ 22.3
a < h
not possible

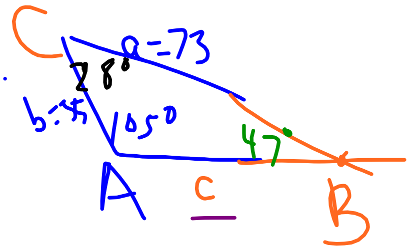
No solution

EXAMPLE 3 The Ambiguous Case—One or No Solution

B. Find all solutions for the given triangle, if possible. If no solution exists, write no solution. Round side lengths to the nearest tenth and angle measures to the nearest degree.
$A = 105°$, $a = 73$, $b = 55$

A is obtuse
$a > b$
$73 > 55$
1 solution

3b.



$$\frac{\sin 105°}{73} = \frac{\sin B}{55}$$

$$\sin B = \frac{55\sin 105°}{73}$$

$$B = \sin^{-1}\left(\frac{55\sin 105°}{73}\right)$$

$$B \approx 47°$$

$$C = 180 - (105 + 47)$$

$$C \approx 28°$$

$$\frac{\sin 105°}{73} = \frac{\sin 28°}{c}$$

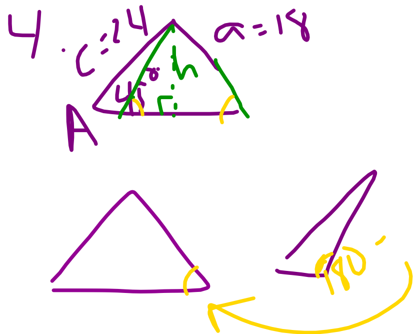
$$c = \frac{73\sin 28°}{\sin 105°}$$

$$c \approx 35.5$$

EXAMPLE 4 The Ambiguous Case-Two Solutions

Find two triangles for which $A = 45°$, $a = 18$, and $c = 24$. Round side lengths to the nearest tenth and angle measures to the nearest degree.

See next slide for work space.



Acute ∠A

$a < c$

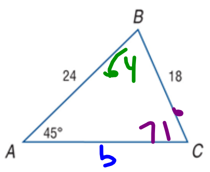
$18 < 24$

find h

$h = 24 \sin 45°$

$h \approx 17.7$

Solution 1 $\angle C$ is acute.



$\frac{\sin 45°}{18} = \frac{\sin C}{24}$

$\sin C = \frac{24 \sin 45°}{18}$

$C = \sin^{-1}\left(\frac{24 \sin 45°}{18}\right)$

$C \approx 71°$

$B = 180 - (71° + 45°)$

$B \approx \underline{64°}$

$\frac{\sin 45°}{18} = \frac{\sin 64°}{b}$

$b = \frac{18 \sin 64°}{\sin 45°}$

$b \approx 22.9$

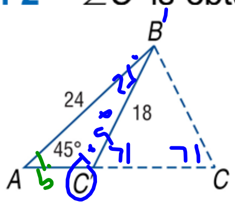
Case 1

$C \approx 71°$

$B = 64°$

$b \approx 22.9$

Solution 2 $\angle C'$ is obtuse.



$C' = 180 - 71$

$C' = 109°$

$B' = 180 - (45 + 109)$

$B' = 26°$

$\frac{\sin 45°}{18} = \frac{\sin 26°}{b}$

$b' = \frac{18 \sin 26°}{\sin 45°}$

$b' \approx \underline{11.2}$

Case 2

$C' \approx 109°$

$B' \approx 26°$

$b' \approx 11.2$