2 Damped Trigonometric Functions When a sinusoidal function is multiplied by another function $f(x)$, the graph of their product oscillates between the graphs of $y = f(x)$ and $y = -f(x)$. When this product reduces the amplitude of the wave of the original sinusoid, it is called **damped oscillation**, and the product of the two functions is known as a **damped trigonometric function**. This change in oscillation can be seen in Figures 4.5.3 and 4.5.4 for the graphs of $y = \sin x$ and $y = 2x \sin x$.

$y = 2x$
$y = -2x$

$y = f(x)$
$y = -f(x)$
$y = f(x) \sin x$

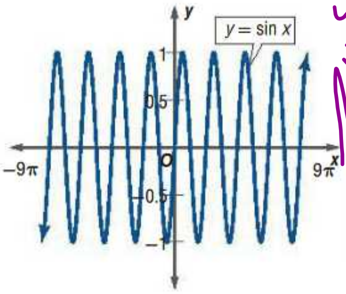


Figure 4.5.3

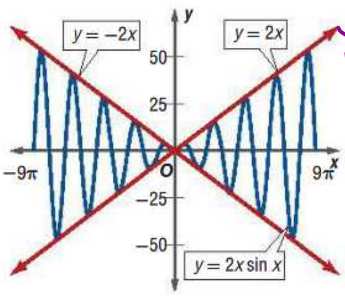


Figure 4.5.4

StudyTip

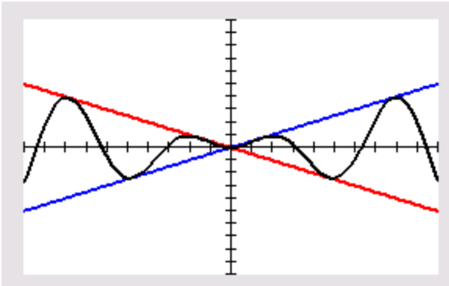
Damped Functions Trigonometric functions that are multiplied by constants do not experience damping. The constant affects the amplitude of the function.

A damped trigonometric function is of the form $y = f(x) \sin bx$ or $y = f(x) \cos bx$, where $f(x)$ is the **damping factor**.

Damped oscillation occurs as x approaches $\pm\infty$ or as x approaches 0 from both directions.

EXAMPLE 5 Sketch Damped Trigonometric Functions

A. Identify the damping factor $f(x)$ of $y = \frac{x}{2}\sin x$. Then use a graphing calculator to sketch the graphs of $f(x)$, $-f(x)$, and the given function in the same viewing window. Describe the behavior of the graph.



5a. $y = \frac{x}{2}\sin x$

$f(x) = \frac{x}{2}$ or $\frac{1}{2}x$

$y_1 = \frac{x}{2}$

$y_2 = -\frac{x}{2}$

$y = \frac{x}{2}\sin x$

The amplitude is decreasing as x approaches 0 from both directions

EXAMPLE 5 Sketch Damped Trigonometric Functions

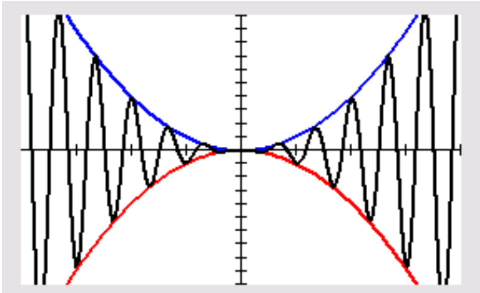
B. Identify the damping factor $f(x)$ of $y = x^2 \cos 3x$. Then use a graphing calculator to sketch the graphs of $f(x)$, $-f(x)$, and the given function in the same viewing window. Describe the behavior of the graph.

So $y = x^2 \cos 3x$

$f(x) = x^2$
damping factor

$y_1 = x^2$
$y_2 = -x^2$
$y_3 = x^2 \cos 3x$

WINDOW
Xmin=-12.56637061
Xmax=12.56637061
Xscl=3.1415926535898
Ymin=-100
Ymax=100
Yscl=10



The amplitude is decreasing as x approaches 0 from both sides.

When the amplitude of the motion of an object decreases with time due to friction, the motion is called *damped harmonic motion*.

$-ct$

Key Concept **Damped Harmonic Motion**

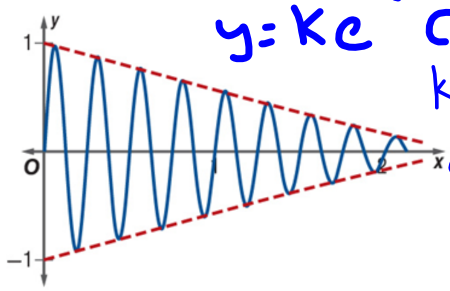
$a(t) = ke^{-ct}$

$y = ke^{-ct} \cos \omega t$

Words An object is in **damped harmonic motion** when the amplitude is determined by the function $a(t) = ke^{-ct}$.

Symbols For $y = ke^{-ct} \sin \omega t$ and $y = ke^{-ct} \cos \omega t$, where $c > 0$, k is the displacement, c is the damping constant, t is time, and ω is the period.

Model



k is displacement

c is damping ~~friction~~ constant

t: time

period: $\frac{2\pi}{\omega}$

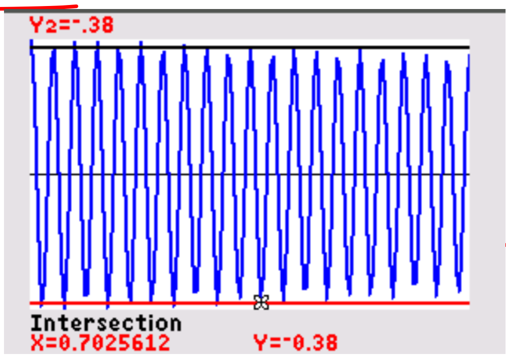
($\cos bx$ per $\frac{2\pi}{b}$)

The greater the damping constant c, the faster the amplitude approaches 0. The magnitude of c depends on the size of the object and the material of which it is composed.

Real-World EXAMPLE 6 Damped Harmonic Motion

A. **MUSIC** A guitar string is plucked at a distance of 0.95 centimeters above its rest position, then released, causing a vibration. The damping constant for the string is 1.3, and the note produced has a frequency of 200 cycles per second. Write a trigonometric function that models the motion of the string.

WINDOW
Xmin=0.65
Xmax=0.75
Xscl=1
Ymin=-0.4
Ymax=0.4
Yscl=1



$y = ke^{-ct}\cos \omega t$

$k = .95$ $\quad f = \frac{1}{p}$ $\quad p = \frac{2\pi}{\omega}$

$c = 1.3$ $\quad 200 = \frac{1}{p}$ $\quad \frac{1}{200} = \frac{2\pi}{\omega}$

$p = \frac{1}{200}$ $\quad \omega = 400\pi$

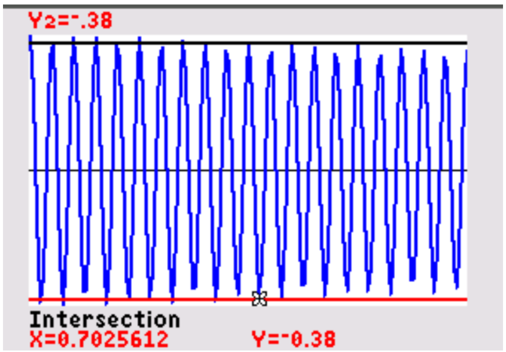
$y = ke^{-ct}\cos \omega t$

$y = .95e^{-1.3t}\cos 400\pi t$

Real-World EXAMPLE 6 Damped Harmonic Motion

B. MUSIC A guitar string is plucked at a distance of 0.95 centimeters above its rest position, then released, causing a vibration. The damping constant for the string is 1.3, and the note produced has a frequency of 200 cycles per second. Determine the amount of time t that it takes the string to be damped so that $-0.38 \le y \le 0.38$.

```
WINDOW
 Xmin=0.65
 Xmax=0.75
 Xscl=1
 Ymin=-0.4
 Ymax=0.4
 Yscl=1
```



about 0.7 seconds

EXAMPLE 1 **Guided Practice**

A. Locate the vertical asymptotes of $y = \tan 4x$.

A. vertical asymptotes: $x = \frac{n\pi}{4}$, n is an odd integer

B. vertical asymptotes: $x = \frac{n\pi}{2}$, n is an odd integer

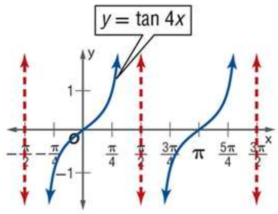
C. vertical asymptotes: $x = \frac{n\pi}{4}$, n is an integer

D. vertical asymptotes: $x = \frac{n\pi}{8}$, n is an odd integer

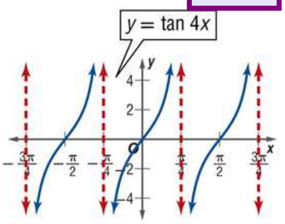
EXAMPLE 1 **Guided Practice**

B. **Sketch the graph of $y = \tan 4x$.**

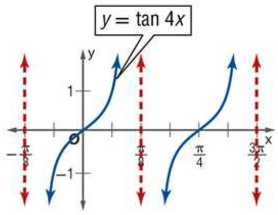
A.



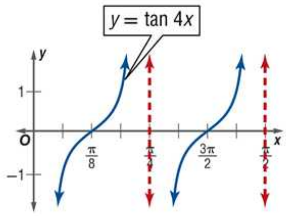
C.



B.



D.



EXAMPLE 2 **Guided Practice**

Locate the vertical asymptotes of the graph of $y = -\tan(3x + \pi)$.

A. vertical asymptotes: $x = \frac{n\pi}{6}$, ***n*** is an odd integer

B. vertical asymptotes: $x = \frac{n\pi}{3}$, ***n*** is an integer

C. vertical asymptotes: $x = \frac{n\pi}{3}$, ***n*** is an odd integer

D. vertical asymptotes: $x = \frac{n\pi}{6}$, ***n*** is an integer

EXAMPLE 3 **Guided Practice**

A. Locate the vertical asymptotes of $y = \frac{1}{2}\cot 2x$.

A. vertical asymptotes: $x = \frac{n\pi}{2}$, n is an integer

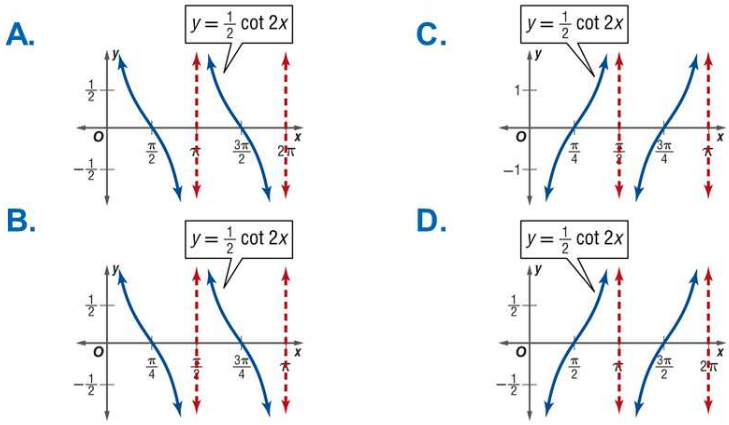
B. vertical asymptotes: $x = \frac{n\pi}{2}$, n is an integer

C. vertical asymptotes: $x = n\pi$, n is an integer

D. vertical asymptotes: $x = n\pi$, n is an integer

EXAMPLE 3 Guided Practice

B. Sketch the graph of $y = \frac{1}{2}\cot 2x$.



EXAMPLE 4 **Guided Practice**

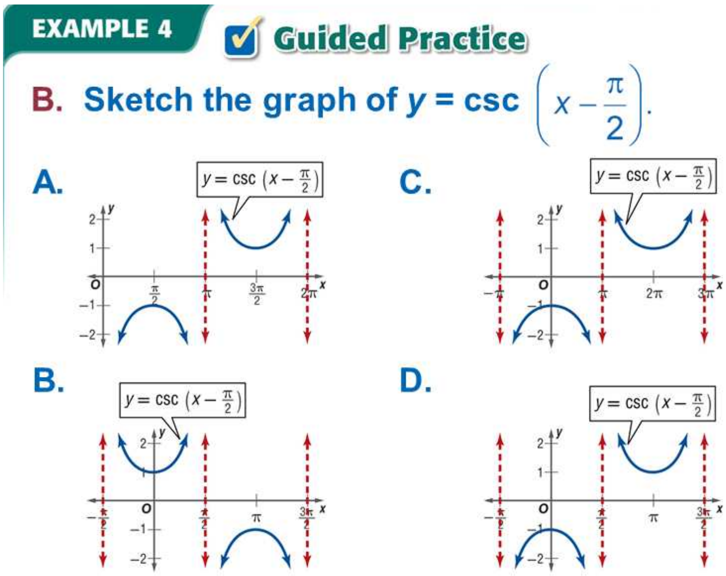
A. Locate the vertical asymptotes of $y = \csc\left(x - \frac{\pi}{2}\right)$.

A. $x = n\pi$, n is an odd integer

B. $x = \frac{n\pi}{2}$, n is an odd integer

C. $x = \frac{n\pi}{2}$, n is an odd integer

D. $x = n\pi$, n is an integer


EXAMPLE 4
Guided Practice
B. Sketch the graph of $y = \csc\left(x - \frac{\pi}{2}\right)$.
A.
$y = \csc\left(x - \frac{\pi}{2}\right)$
C.
$y = \csc\left(x - \frac{\pi}{2}\right)$
B.
$y = \csc\left(x - \frac{\pi}{2}\right)$
D.
$y = \csc\left(x - \frac{\pi}{2}\right)$

EXAMPLE 5 **Guided Practice**

Identify the damping factor $f(x)$ of $y = 4x \sin x$.

A. $f(x) = 4x$

B. $f(x) = \frac{1}{4x}$

C. $f(x) = \sin x$

D. $f(x) = 4x \sin x$

Real-World EXAMPLE 6 Guided Practice

MUSIC Suppose another string on the guitar was plucked 0.3 centimeter above its rest position with a frequency of 64 cycles per second and a damping constant of 1.4. Write a trigonometric function that models the motion of the string y as a function of time t.

A. $y = 0.3e^{-2.8t} \cos 64\pi t$

B. $y = 0.6e^{-0.07t} \cos 32\pi t$

C. $y = 0.15e^{-5.6t} \cos 256\pi t$

D. $y = 0.3e^{-1.4t} \cos 128\pi t$

EXAMPLE 1 Guided Practice

A. Locate the vertical asymptotes of $y = \tan 4x$.

A. vertical asymptotes: $x = \frac{n\pi}{4}$, n is an odd integer

B. vertical asymptotes: $x = \frac{n\pi}{2}$, n is an odd integer

C. vertical asymptotes: $x = \frac{n\pi}{4}$, n is an integer

D. vertical asymptotes: $x = \frac{n\pi}{8}$, n is an odd integer

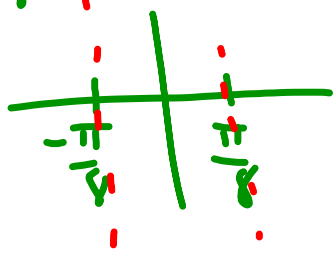
$\boxed{D}$

$4x = -\frac{\pi}{2}$

$x = -\frac{\pi}{8}$

$4x = \frac{\pi}{2}$

$x = \frac{\pi}{8}$



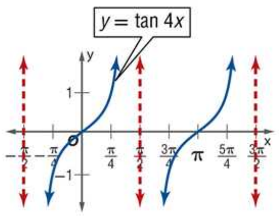
$x = \frac{\pi}{8} n$

EXAMPLE 1 Guided Practice

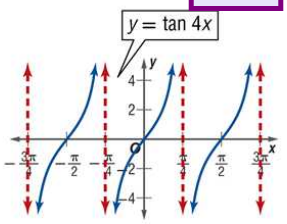
B. Sketch the graph of $y = \tan 4x$.

B

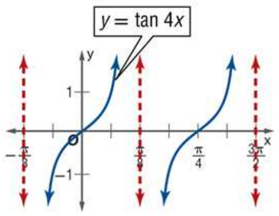
A.



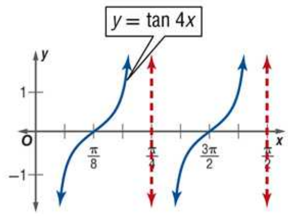
C.



B.



D.



EXAMPLE 2 Guided Practice

Locate the vertical asymptotes of the graph of $y = -\tan(3x + \pi)$.

$\boxed{A}$

A. vertical asymptotes: $x = \frac{n\pi}{6}$, n is an odd integer

B. vertical asymptotes: $x = \frac{n\pi}{3}$, n is an integer

C. vertical asymptotes: $x = \frac{n\pi}{3}$, n is an odd integer

D. vertical asymptotes: $x = \frac{n\pi}{6}$, n is an integer

$3x + \pi = -\frac{\pi}{2}$

$3x = -\frac{\pi}{2} - \frac{2\pi}{2}$

$\frac{1}{3} \cdot 3x = -\frac{3\pi}{2} \cdot \frac{1}{3}$

$x = -\frac{3\pi}{6}$

$3x + \pi = \frac{\pi}{2}$

$-\pi \quad -\frac{2\pi}{2}$

$\frac{1}{3} \cdot 3x = -\frac{\pi}{2} \cdot \frac{1}{3}$

$x = -\frac{\pi}{6}$

EXAMPLE 3 **Guided Practice**

A. Locate the vertical asymptotes of $y = \frac{1}{2}\cot 2x$.

$\boxed{B}$

A. vertical asymptotes: $x = \frac{n\pi}{2}$, n is an integer

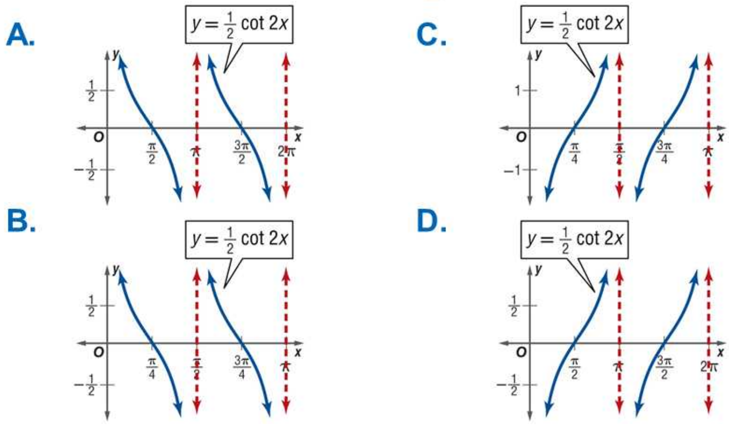
B. vertical asymptotes: $x = \frac{n\pi}{2}$, n is an integer

C. vertical asymptotes: $x = n\pi$, n is an integer

D. vertical asymptotes: $x = n\pi$, n is an integer

EXAMPLE 3 **Guided Practice**

B. Sketch the graph of $y = \dfrac{1}{2}\cot 2x$.



B

EXAMPLE 4 **Guided Practice**

A. Locate the vertical asymptotes of $y = \csc\left(x - \frac{\pi}{2}\right)$.

A. $x = n\pi$, n is an odd integer

B or C (same ans)

B. $x = \frac{n\pi}{2}$, n is an odd integer

C. $x = \frac{n\pi}{2}$, n is an odd integer

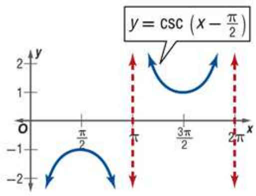
D. $x = n\pi$, n is an integer

EXAMPLE 4 **Guided Practice**

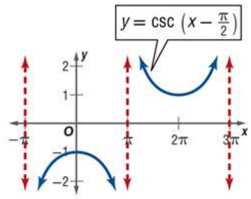
B. Sketch the graph of $y = \csc\left(x - \frac{\pi}{2}\right)$.

$\boxed{D}$

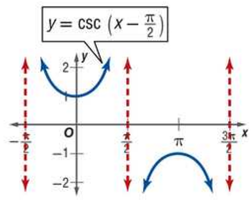
A.



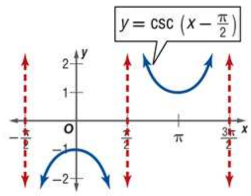
C.



B.



D.



EXAMPLE 5 **Guided Practice**

Identify the damping factor $f(x)$ of $y = 4x \sin x$.

A

A. $f(x) = 4x$

B. $f(x) = \dfrac{1}{4x}$

C. $f(x) = \sin x$

D. $f(x) = 4x \sin x$

Real-World EXAMPLE 6 **Guided Practice**

MUSIC Suppose another string on the guitar was plucked 0.3 centimeter above its rest position with a frequency of 64 cycles per second and a damping constant of 1.4. Write a trigonometric function that models the motion of the string y as a function of time t.

A. $y = 0.3e^{-2.8t} \cos 64\pi t$

B. $y = 0.6e^{-0.07t} \cos 32\pi t$

C. $y = 0.15e^{-5.6t} \cos 256\pi t$

D. $y = 0.3e^{-1.4t} \cos 128\pi t$

$\boxed{D}$

$y = ke^{-ct} \cos \omega t$

$k = .3$

$F = 64$ $\quad p = \frac{2\pi}{\omega}$ $\quad c = 1.4$

$F = \frac{1}{p}$ $\quad \frac{1}{64} = \frac{2\pi}{\omega}$

$64 = \frac{1}{p}$ $\quad \omega = 128\pi$

$p = \frac{1}{64}$

$y = ke^{-ct} \cos \omega t$

$y = .3e^{-1.4t} \cos 128\pi t$

4-5 Graphing Other Trigonometric Functions

EVEN PAGE

TOC

EQ: Can you graph tangent and reciprocal trigonometric functions and graph damped trigonometric functions?

How are you doing? Write answer next to Essential Question
1. I don't understand the material
2. I understand a little.
3. I understand this material.
4. I could teach this to someone

Summary: At least 3 sentences...