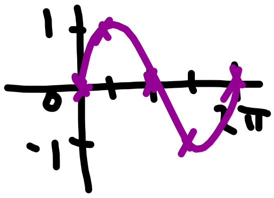
4-5 Graphing Other Trigonometric Functions

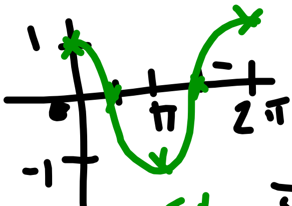
EVEN PAGE

TOC

EQ: Can you graph tangent and reciprocal trigonometric functions and graph damped trigonometric functions?



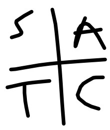
$y = \sin x$



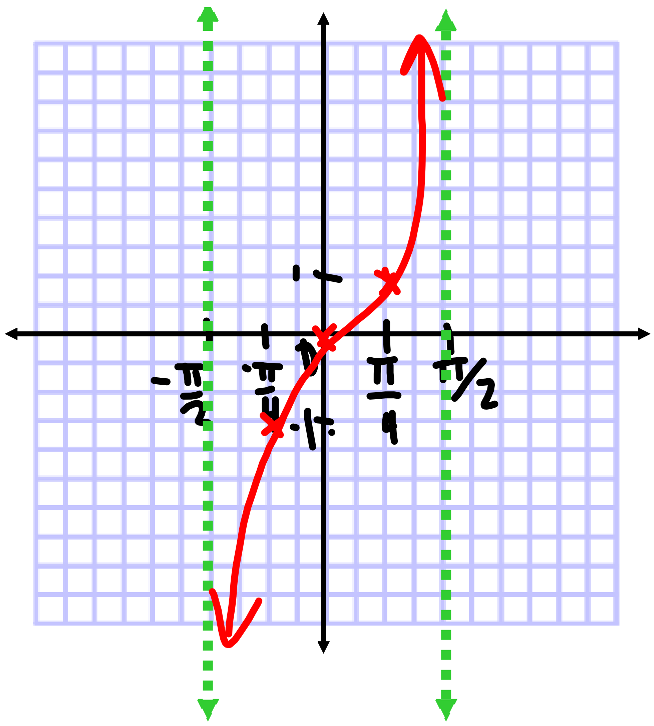
$y = \cos x$

How are you d
1. I don't unde
2. I understan
3. I understan
4. I could teac

rad	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
deg	0	30	45	60	90
sin	0	$\frac{1}{2}$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$\frac{1}{2}$	0
tan	0	$\sqrt{3}/3$	1	$\sqrt{3}$	und



Summary: At least 3 sentences...



$f(x) = \tan x$

period π

starts $-\frac{\pi}{2}$ ends $\frac{\pi}{2}$

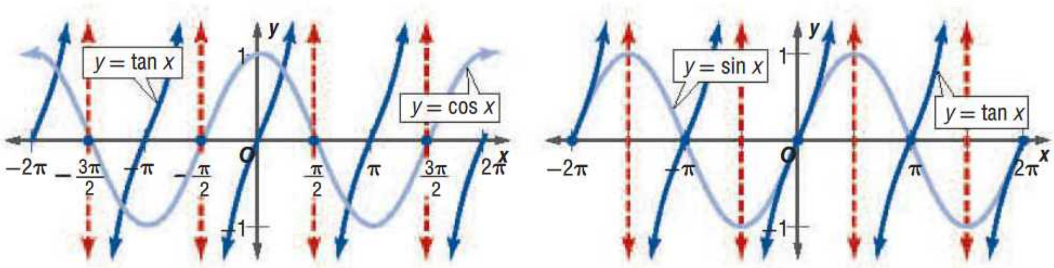
asymptotes

$x = \frac{\pi}{2} + \pi n \quad n \in \mathbb{Z}$

$R: (-\infty, \infty)$

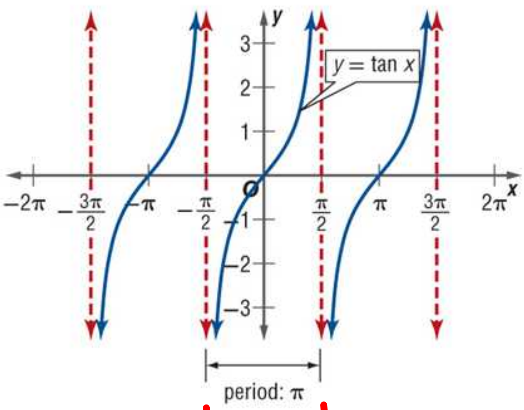
1 Tangent and Reciprocal Functions In Lesson 4-4, you graphed the sine and cosine functions on the coordinate plane. You can use the same techniques to graph the tangent function and the reciprocal trigonometric functions—cotangent, secant, and cosecant.

Since $\tan x = \frac{\sin x}{\cos x}$, the tangent function is undefined when $\cos x = 0$. Therefore, the tangent function has a *vertical asymptote* whenever $\cos x = 0$. Similarly, the tangent and sine functions each have zeros at integer multiples of π because $\tan x = 0$ when $\sin x = 0$.



Key Concept Properties of the Tangent Function

Domain:	$x \in \mathbb{R}, x \neq \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$
Range:	$(-\infty, \infty)$
***x*-intercepts:**	$n\pi, n \in \mathbb{Z}$ $0+\pi n$
***y*-intercept:**	0
Continuity:	infinite discontinuity at $x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$
Asymptotes:	$x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$
Symmetry:	origin (odd function)
Extrema:	none
End Behavior:	$\lim\limits_{x \to -\infty} \tan x$ and $\lim\limits_{x \to \infty} \tan x$ do not exist. The function oscillates between $-\infty$ and ∞.



Period π

The general form of the tangent function, which is similar to that of the sinusoidal functions, is $y = a \tan (bx + c) + d$, where a produces a vertical stretch or compression, b affects the period, c produces a phase shift, d produces a vertical shift and neither a or b are 0.

StudyTip

Alternate Method When graphing a function with only a horizontal translation c, you can find the key points by adding c to each of the x-coordinates of the key points of the parent function.

Key Concept Period of the Tangent Function

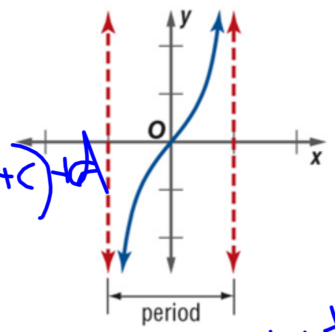
Words The *period* of a tangent function is the distance between any two consecutive vertical asymptotes.

Symbols For $y = a \tan (bx + c)$, where $b \neq 0$, period $= \frac{\pi}{|b|}$.

$y = a\tan(bx+c)+d$

period $= \frac{\pi}{|b|}$

Model



asymptotes begin $bx+c = -\frac{\pi}{2}$

end asymptote $bx+c = \frac{\pi}{2}$

StudyTip

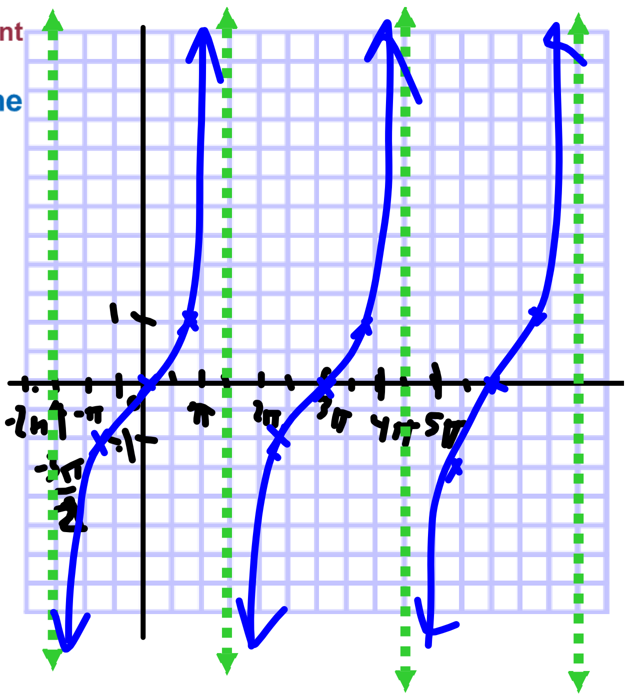
Amplitude The term *amplitude* does not apply to the tangent or cotangent functions because the heights of these functions are infinite.

Two consecutive vertical asymptotes for $y = \tan x$ are $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. You can find two consecutive vertical asymptotes for any tangent function of the form $y = a \tan (bx + c) + d$ by solving the equations $\boxed{bx + c = -\frac{\pi}{2}}$ and $\boxed{bx + c = \frac{\pi}{2}}$.

You can sketch the graph of a tangent function by plotting the vertical asymptotes, x-intercepts, and points between the asymptotes and x-intercepts.

EXAMPLE 1 Graph Horizontal Dilations of the Tangent Function

Locate the vertical asymptotes, and sketch the graph of $y = \tan \frac{x}{3}$.



$y = \tan\frac{x}{3}$

$y = \tan\frac{1}{3}x$

$y = \tan bx$

$b = \frac{1}{3}$ period $= \frac{\pi}{\frac{1}{3}} = \pi \cdot \frac{3}{1} = 3\pi$

begins ends

$3 \cdot \frac{x}{3} = -\frac{\pi}{2} \cdot \frac{3}{1}$ $3 \cdot \frac{x}{3} = \frac{\pi}{2} \cdot 3$

$x = -\frac{3\pi}{2}$ $x = \frac{3\pi}{2}$

EXAMPLE 2 Graph Reflections and Translations of the Tangent Function

A. Locate the vertical asymptotes, and sketch the graph of $y = -\tan\frac{x}{4}$.

2a. $y = -\tan\frac{x}{4} = -\tan\left(\frac{1}{4}x\right)$

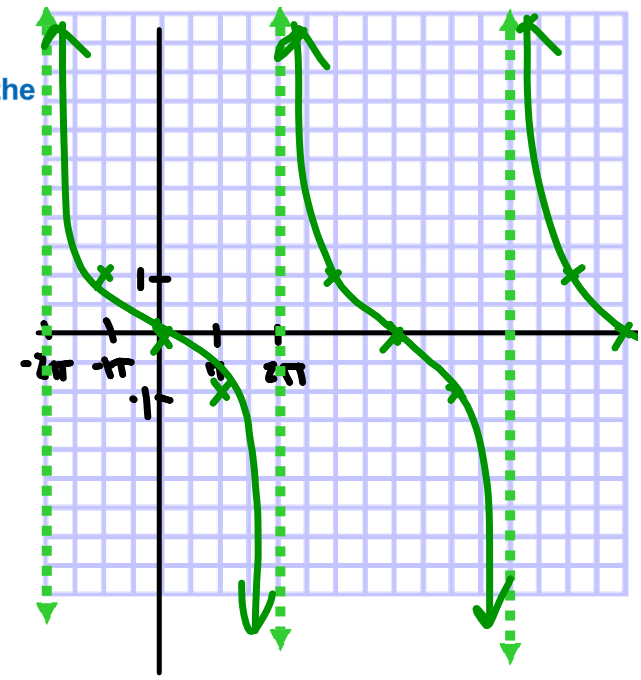
reflectin x-axis

$b = \frac{1}{4}$

period $= \frac{\pi}{\frac{1}{4}} = 4\pi$

begin: $\frac{x}{4} = -\frac{\pi}{2}$, $x = -2\pi$

end: $\frac{x}{4} = \frac{\pi}{2}$, $x = 2\pi$



EXAMPLE 2 Graph Reflections and Translations of the Tangent Function

2b.

B. Locate the vertical asymptotes, and sketch the graph of $y = -\tan\left(x + \frac{\pi}{2}\right)$.

$y = -\tan\left(x + \frac{\pi}{2}\right)$

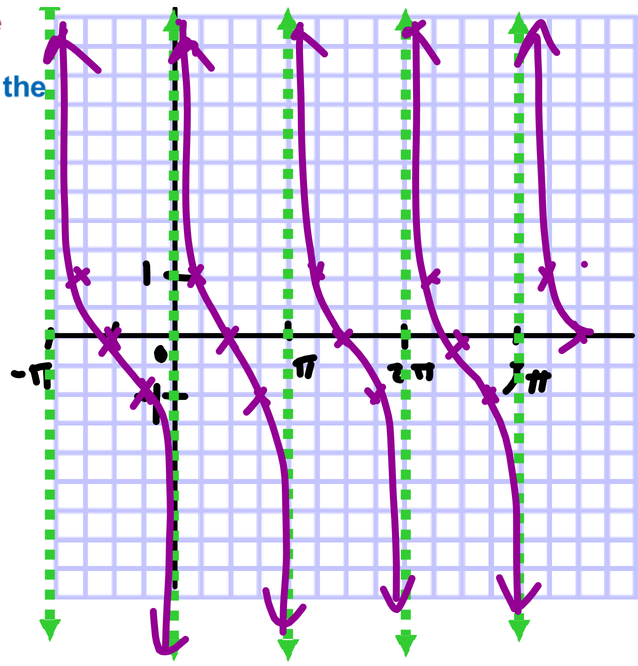
reflected in x-axis

$b = 1$ $\quad c = \frac{\pi}{2}$ horiz. shift $\frac{-c}{|b|} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2}$

left asymp. $\quad$ right asymp.

$x + \frac{\pi}{2} = -\frac{\pi}{2}$ $\quad$ $x + \frac{\pi}{2} = \frac{\pi}{2}$

$x = -\pi$ $\quad$ $x = 0$



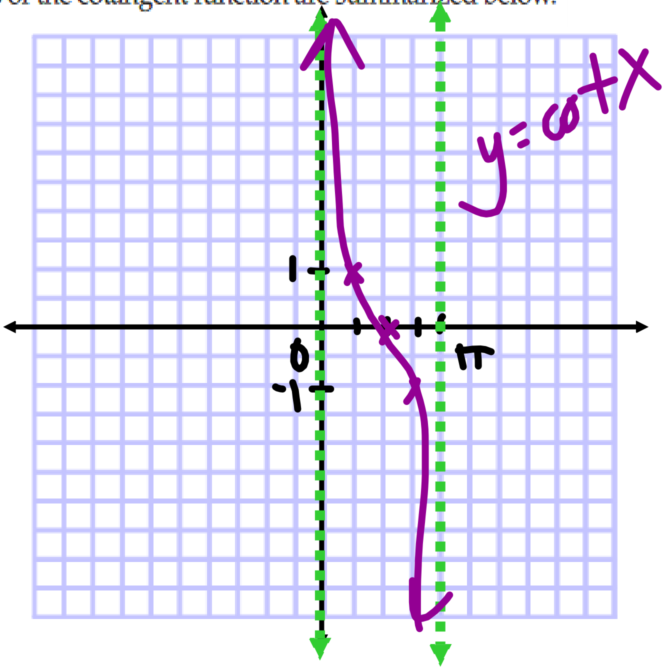
The cotangent function is the reciprocal of the tangent function, and is defined as $\cot x = \frac{\cos x}{\sin x}$. Like the tangent function, the period of a cotangent function of the form $y = a\cot(bx + c) + d$ can be found by calculating $\frac{\pi}{|b|}$. Two consecutive vertical asymptotes can be found by solving the equations $bx + c = 0$ and $bx + c = \pi$. The properties of the cotangent function are summarized below.

$y = a\cot(bx+c)+d$

period: $\frac{\pi}{|b|}$

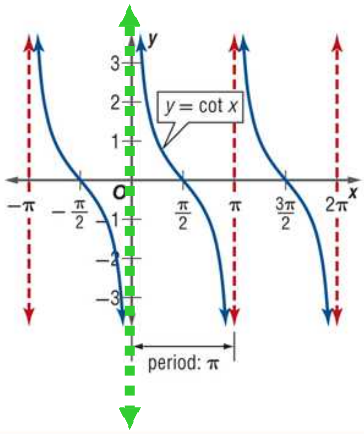
begin asymptote $bx+c=0$

end asymptote $bx+c=\pi$



Key Concept **Properties of the Cotangent Function**

Domain:	$x \in \mathbb{R}, x \neq n\pi, n \in \mathbb{Z}$
Range:	$(-\infty, \infty)$ $x \neq 0 + \pi n$
x-intercepts:	$\frac{\pi}{2} + n\pi, n \in \mathbb{Z}$
y-intercept:	none
Continuity:	infinite discontinuity at $x = n\pi, n \in \mathbb{Z}$
Asymptotes:	$x = n\pi, n \in \mathbb{Z}$ $x: 0 + n\pi$
Symmetry:	origin (odd function)
Extrema:	none
End Behavior:	$\lim_{x \to -\infty} \cot x$ and $\lim_{x \to \infty} \cot x$ do not exist. The function oscillates between $-\infty$ and ∞.



Technology Tip

Graphing a Cotangent Function When using a calculator to graph a cotangent function, enter the reciprocal of tangent, $y = \frac{1}{\tan x}$. Graphing calculators may produce solid lines where the asymptotes occur. Setting the mode to DOT will eliminate the line.

You can sketch the graph of a cotangent function using the same techniques that you used to sketch the graph of a tangent function.

EXAMPLE 3 Sketch the Graph of a Cotangent Function

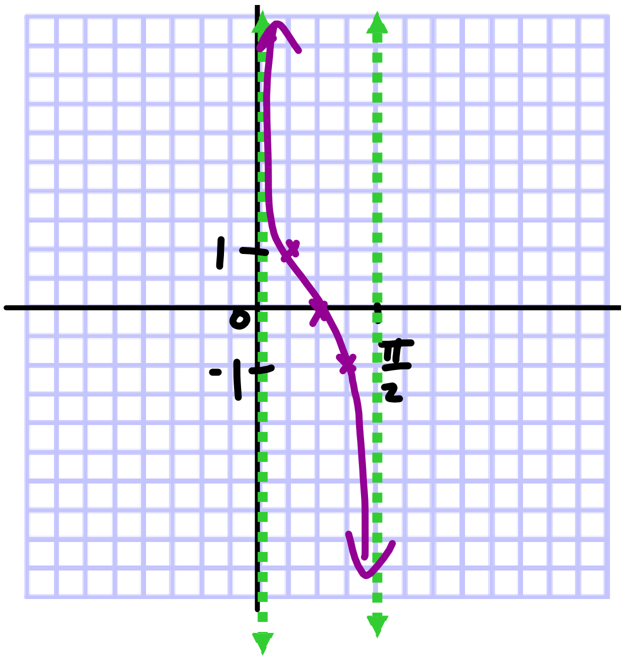
Locate the vertical asymptotes, and sketch the graph of $y = \cot 2x$.

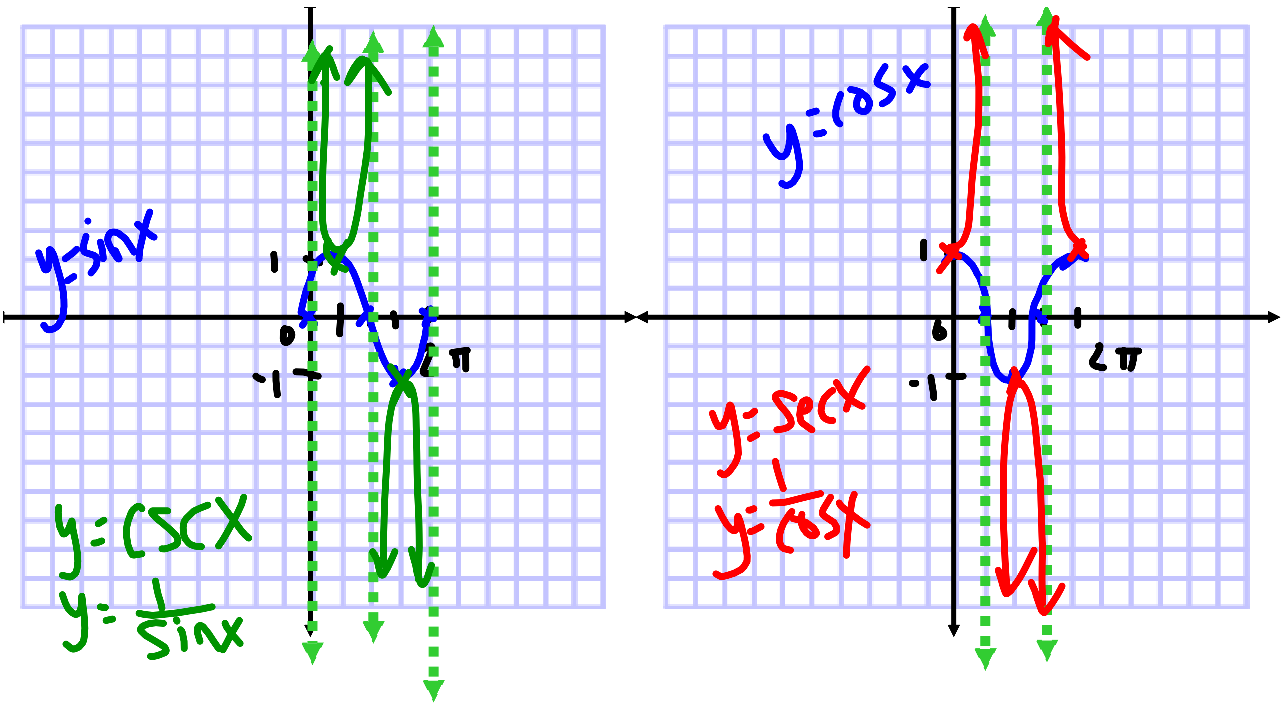
$y = \cot 2x$

period: $\frac{\pi}{2}$

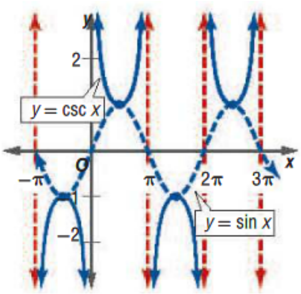
begin: $2x = 0$, $x = 0$

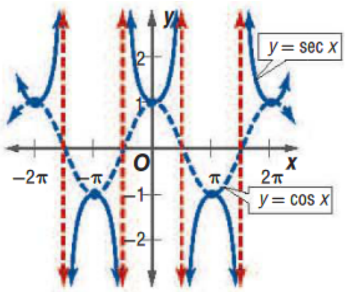
end: $2x = \pi$, $x = \frac{\pi}{2}$




y=sinx
1
0
-1
2π
y=cscx
y= 1/sinx
y=cosx
1
0
-1
2π
y=secx
y= 1/cosx

The reciprocals of the sine and cosine functions are defined as $\csc x = \frac{1}{\sin x}$ and $\sec x = \frac{1}{\cos x}$, as shown below.



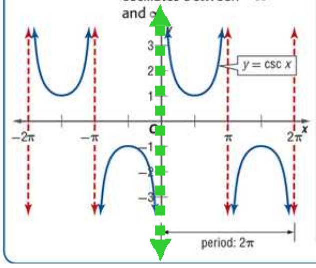


The cosecant function has asymptotes when $\sin x = 0$, which occurs at integer multiples of π. Likewise, the secant function has asymptotes when $\cos x = 0$, located at odd multiples of $\frac{\pi}{2}$. Notice also that the graph of $y = \csc x$ has a relative minimum at each maximum point on the sine curve, and a relative maximum at each minimum point on the sine curve. The same is true for the graphs of $y = \sec x$ and $y = \cos x$.

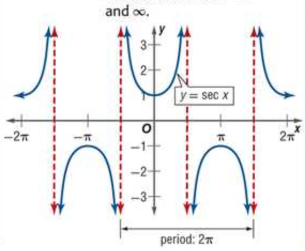
The properties of the cosecant and secant functions are summarized below.

Key Concept Properties of the Cosecant and Secant Functions

Cosecant Function	
Domain:	$x \in \mathbb{R}, x \neq n\pi, n \in \mathbb{Z}$
Range:	$(-\infty, -1]$ and $[1, \infty)$
***x*-intercepts:**	none
***y*-intercept:**	none
Continuity:	infinite discontinuity at $x = n\pi, n \in \mathbb{Z}$
Asymptotes:	$x = n\pi, n \in \mathbb{Z}$
Symmetry:	origin (odd function)
End Behavior:	$\lim_{x \to -\infty} \csc x$ and $\lim_{x \to \infty} \csc x$ do not exist. The function oscillates between $-\infty$ and ∞.



Secant Function	
Domain:	$x \in \mathbb{R}, x \neq \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$
Range:	$(-\infty, -1]$ and $[1, \infty)$
***x*-intercepts:**	none
***y*-intercept:**	1
Continuity:	infinite discontinuity at $x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$
Asymptotes:	$x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$
Symmetry:	y-axis (even function)
Behavior:	$\lim_{x \to -\infty} \sec x$ and $\lim_{x \to \infty} \sec x$ do not exist. The function oscillates between $-\infty$ and ∞.



TechnologyTip

Graphing Graphing the cosecant and secant functions on a calculator is similar to graphing the cotangent function. Enter the reciprocals of the sine and cosine functions.

StudyTip

Finding Asymptotes and Key Points You can use the periodic nature of trigonometric graphs to help find asymptotes and key points. In Example 4a, notice that the vertical asymptote $x = -\frac{\pi}{2}$ is equidistant from the calculated asymptotes, $x = -\frac{3\pi}{2}$ and $x = \frac{\pi}{2}$.

Like the sinusoidal functions, the period of a secant function of the form $y = a \sec (bx + c) + d$ or cosecant function of the form $y = a \csc (bx + c) + d$ can be found by calculating $\frac{2\pi}{|b|}$. Two vertical asymptotes for the secant function can be found by solving the equations $bx + c = -\frac{\pi}{2}$ and $bx + c = \frac{3\pi}{2}$ and two vertical asymptotes for the cosecant function can be found by solving $bx + c = -\pi$ and $bx + c = \pi$.

To sketch the graph of a cosecant or secant function, locate the asymptotes of the function and find the corresponding relative maximum and minimums points.

EXAMPLE 4 Sketch Graphs of Cosecant and Secant Functions

A. Locate the vertical asymptotes, and sketch the graph of $y = -\sec 2x$.

4a $y = -\sec 2x$

reflect x-axis

period: $\frac{2\pi}{2} = \pi$

~~Start~~

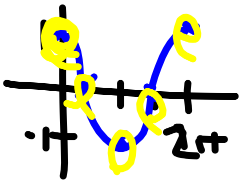
$2x = \frac{\pi}{2}$

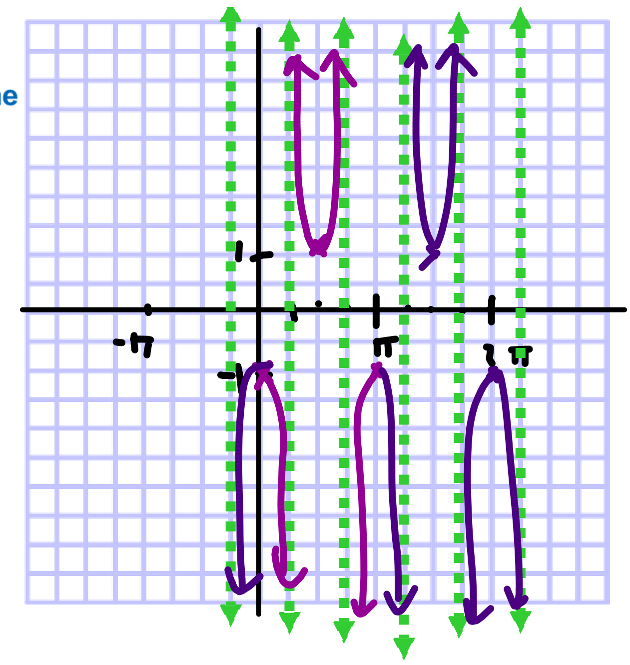
$x = \frac{\pi}{4}$

$2x = \frac{3\pi}{2}$

$x = \frac{3\pi}{4}$

asymptote



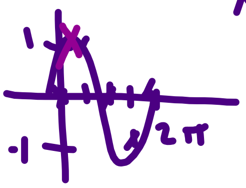


EXAMPLE 4 Sketch Graphs of Cosecant and Secant Functions

B. Locate the vertical asymptotes, and sketch the graph of $y = \csc\left(x + \frac{\pi}{3}\right)$.

4b. $y = \csc\left(x + \frac{\pi}{3}\right)$

$\csc x = \frac{1}{\sin x}$



start

$x + \frac{\pi}{3} = 0$

$x = -\frac{\pi}{3}$

end

$x + \frac{\pi}{3} = \frac{2\pi}{1}$

$x = \frac{6\pi}{3} - \frac{\pi}{3}$

$x = \frac{5\pi}{3}$