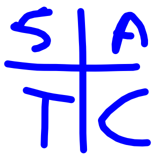
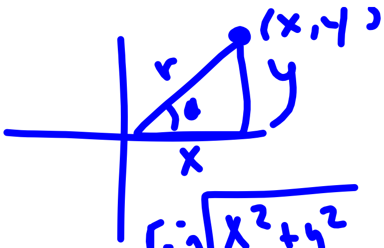
The following slides show things from Notes 4-3 that will help you study for the Chart Quiz. You will be expected to generate the table for special angles of trig functions and answer questions about the +/- signs, and quadrantal angles on the unit circle. You are expected to take the quiz WITHOUT the use of notes or other references.

∠rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
∠deg	0	30	45	60	90
$\sin\theta$	0	$\frac{1}{2}$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$\frac{1}{2}$	0
$\tan\theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	undef

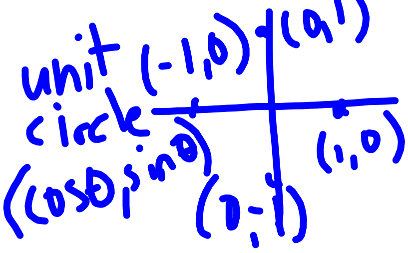




$r = \sqrt{x^2+y^2}$

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$



1 Trigonometric Functions of Any Angle

In Lesson 4-1, the definitions of the six trigonometric functions were restricted to positive acute angles. In this lesson, these definitions are extended to include *any* angle.

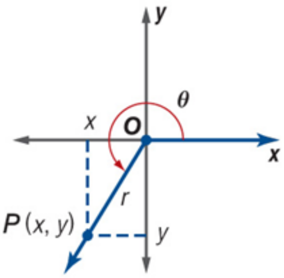
Key Concept — Trigonometric Functions of Any Angle

Let θ be any angle in standard position and point $P(x, y)$ be a point on the terminal side of θ. Let r represent the nonzero distance from P to the origin. That is, let $r = \sqrt{x^2 + y^2} \neq 0$. Then the trigonometric functions of θ are as follows.

$\sin\theta = \frac{y}{r}$ $\quad$ $\csc\theta = \frac{r}{y}, y \neq 0$

$\cos\theta = \frac{x}{r}$ $\quad$ $\sec\theta = \frac{r}{x}, x \neq 0$

$\tan\theta = \frac{y}{x}, x \neq 0$ $\quad$ $\cot\theta = \frac{x}{y}, y \neq 0$



Key Concept Trigonometric Functions of Any Angle

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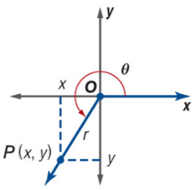
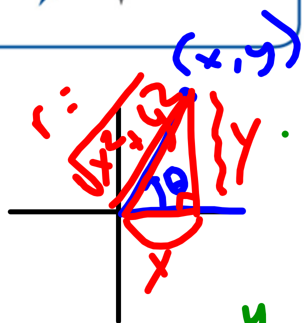
$\tan\theta = \frac{y}{x}, x \neq 0$ $\quad$ $\cot\theta = \frac{x}{y}, y \neq 0$





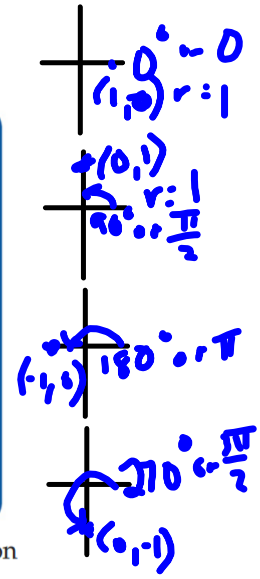
$\sin\theta : \frac{y}{r}$

$\cos\theta : \frac{x}{r}$

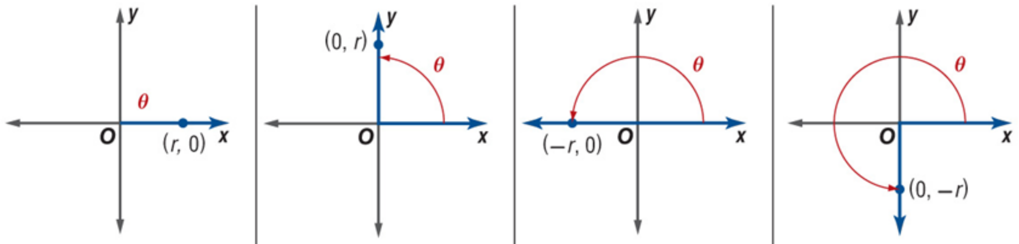
$\tan\theta : \frac{y}{x}$

4-3 Things to Know for Chart Quiz in Math Analysis May 10, 2021

In Example 1, you found the trigonometric values of θ without knowing the measure of θ. Now we will discuss methods for finding these function values when only θ is known. Consider trigonometric functions of quadrantal angles. When the terminal side of an angle θ that is in standard position lies on one of the coordinate axes, the angle is called a **quadrantal angle**.



Key Concept — Common Quadrantal Angles



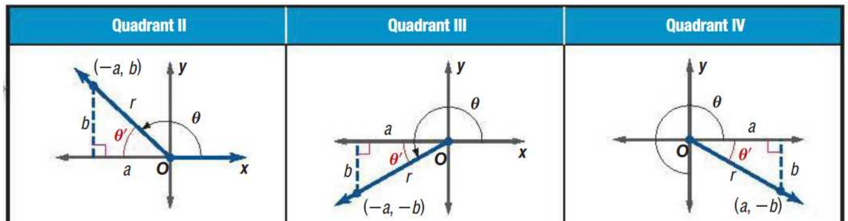
$\theta = 0°$ or 0 radians | $\theta = 90°$ or $\frac{\pi}{2}$ radians | $\theta = 180°$ or π radians | $\theta = 270°$ or $\frac{3\pi}{2}$ radians

You can find the values of the trigonometric functions of quadrantal angles by choosing a point on the terminal side of the angle and evaluating the function at that point. Any point can be chosen. However, to simplify calculations, pick a point for which r equals 1.

StudyTip

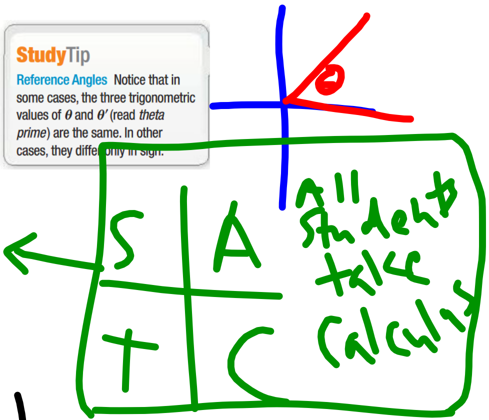
Quadrantal Angles There are infinitely many quadrantal angles that are coterminal with the quadrantal angles listed at the right. The measure of a quadrantal angle is a multiple of 90° or $\frac{\pi}{2}$.

To find the values of the trigonometric functions of angles that are neither acute nor quadrantal, consider the three cases shown below in which a and b are positive real numbers. Compare the values of sine, cosine, and tangent of θ and θ'.

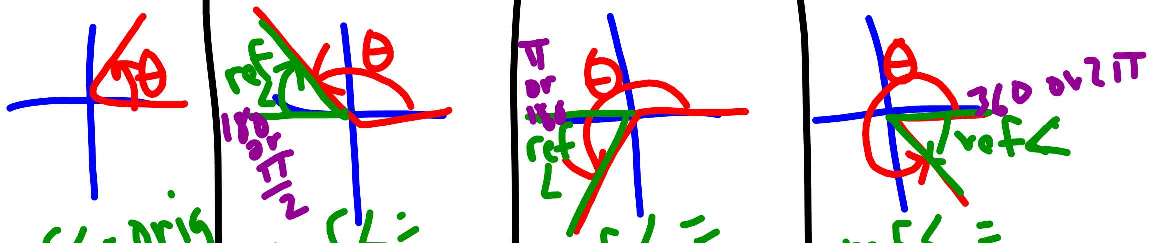


Quadrant II		Quadrant III		Quadrant IV	
$\sin\theta = \frac{b}{r}$	$\sin\theta' = \frac{b}{r}$	$\sin\theta = -\frac{b}{r}$	$\sin\theta' = \frac{b}{r}$	$\sin\theta = -\frac{b}{r}$	$\sin\theta' = \frac{b}{r}$
$\cos\theta = -\frac{a}{r}$	$\cos\theta' = \frac{a}{r}$	$\cos\theta = -\frac{a}{r}$	$\cos\theta' = \frac{a}{r}$	$\cos\theta = \frac{a}{r}$	$\cos\theta' = \frac{a}{r}$
$\tan\theta = -\frac{b}{a}$	$\tan\theta' = \frac{b}{a}$	$\tan\theta = \frac{b}{a}$	$\tan\theta' = \frac{b}{a}$	$\tan\theta = -\frac{b}{a}$	$\tan\theta' = \frac{b}{a}$

StudyTip Reference Angles Notice that in some cases, the three trigonometric values of θ and θ' (read *theta prime*) are the same. In other cases, they differ only in sign.



This angle θ', called a **reference angle**, can be used to find the trigonometric values of any angle θ. ~~To find a reference angle for angles~~ outside the interval $0° < \theta < 360°$ or $0 < \theta < 2\pi$, first find a corresponding coterminal angle in this interval.



ref∠ = orig ∠

ref∠ = 180 − θ or π − θ

ref∠ = θ − 180 or θ − π

ref∠ = 360 − θ or 2π − θ

The signs of the trigonometric functions in each quadrant can be determined using the function definitions given on page 242. For example, because $\sin\theta = \frac{y}{r}$, it follows that $\sin\theta$ is negative when $y < 0$, which occurs in Quadrants III and IV. Using this same logic, you can verify each of the signs for $\sin\theta$, $\cos\theta$, and $\tan\theta$ shown in the diagram. Notice that these values depend only on x and y because r is always positive.

Quadrant II $\sin\theta$: $+$ $\cos\theta$: $-$ $\tan\theta$: $-$	**Quadrant I** $\sin\theta$: $+$ $\cos\theta$: $+$ $\tan\theta$: $+$
Quadrant III $\sin\theta$: $-$ $\cos\theta$: $-$ $\tan\theta$: $+$	**Quadrant IV** $\sin\theta$: $-$ $\cos\theta$: $+$ $\tan\theta$: $-$

x

Ask me about: All Students Take Calculus

Because you know the exact trigonometric values of 30°, 45°, and 60° angles, you can find the exact trigonometric values of *all* angles for which these angles are reference angles. The table lists these values for θ in both degrees and radians.

StudyTip

Memorizing Trigonometric Values

To memorize the exact values of sine for 0°, 30°, 45°, 60°, and 90°, consider the following pattern.

$\sin 0° = \frac{\sqrt{0}}{2}$, or 0

$\sin 30° = \frac{\sqrt{1}}{2}$, or $\frac{1}{2}$

$\sin 45° = \frac{\sqrt{2}}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\sin 90° = \frac{\sqrt{4}}{2}$, or 1

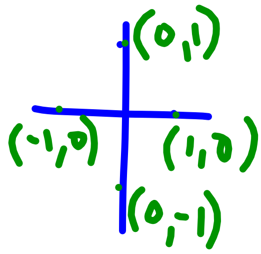
A similar pattern exists for the cosine function, except the values are given in reverse order.

θ	30° or $\frac{\pi}{6}$	45° or $\frac{\pi}{4}$	60° or $\frac{\pi}{3}$
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Ask me about generating a better table

$\angle$ rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\angle$ deg	0	30	45	60	90
$\sin\theta$	0	$\frac{1}{2}$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$\frac{1}{2}$	0
$\tan\theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	undef

$\tan\theta = \frac{\sin\theta}{\cos\theta}$



$r = \sqrt{x^2+y^2}$

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

S	A
T	C

$\angle$ rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\angle$ deg	0	30	45	60	90
$\sin\theta$	0	$\frac{1}{2}$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$\frac{1}{2}$	0
$\tan\theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	undef

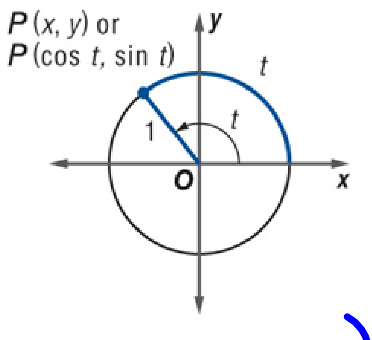
Key Concept Trigonometric Functions on the Unit Circle

Let t be any real number on a number line and let $P(x, y)$ be the point on t when the number line is wrapped onto the unit circle. Then the trigonometric functions of t are as follows.

$\sin t = y$ $\qquad$ $\cos t = x$ $\qquad$ $\tan t = \frac{y}{x}, x \neq 0$

$\csc t = \frac{1}{y}, y \neq 0$ $\qquad$ $\sec t = \frac{1}{x}, x \neq 0$ $\qquad$ $\cot t = \frac{x}{y}, y \neq 0$

Therefore, the coordinates of P corresponding to the angle t can be written as $P(\cos t, \sin t)$.



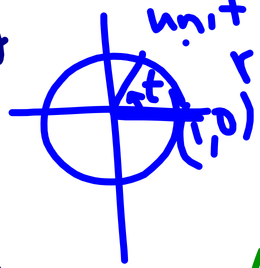
$\sin t = y$ $\quad$ $\csc t = \frac{1}{y}$

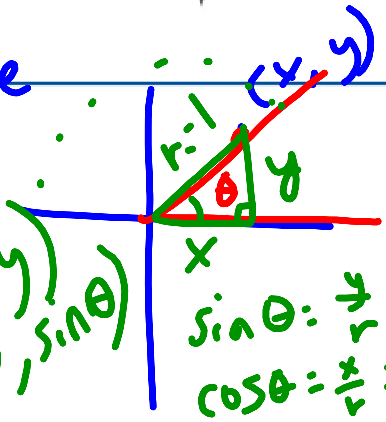
$\cos t = x$ $\quad$ $\sec t = \frac{1}{x}$

$\tan t = \frac{y}{x}$ $\quad$ $\cot t = \frac{x}{y}$

$(x, y) = (\cos t, \sin t)$

unit circle
$r = 1$





(x, y)
$(\cos\theta, \sin\theta)$

$\sin\theta = \frac{y}{r} = y$

$\cos\theta = \frac{x}{r} = x$

4-3 Things to Know for Chart Quiz in Math Analysis

May 10, 2021

Notice that the input value in each of the definitions above can be thought of as an angle measure or as a real number t. When defined as functions of the real number system using the unit circle, the trigonometric functions are often called **circular functions**.

Using reference angles or quadrantal angles, you should now be able to find the trigonometric function values for all integer multiples of 30°, or $\frac{\pi}{6}$ radians, and 45°, or $\frac{\pi}{4}$ radians. These special values wrap to 16 special points on the unit circle, as shown below.

<rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
<deg	0	30	45	60	90
$\sin\theta$	0	$\frac{1}{2}$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$\frac{1}{2}$	0
$\tan\theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	und

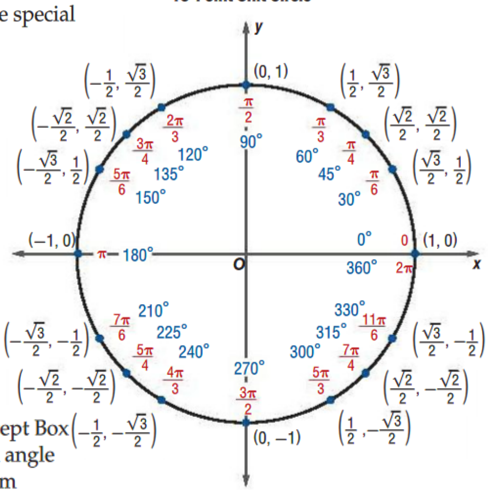
S | A
T | K

unit circle
$(\cos\theta, \sin\theta)$

StudyTip

16-Point Unit Circle You have already memorized these values in the first quadrant. The remaining values can be determined using the x-axis, y-axis, and origin symmetry of the unit circle along with the signs of x and y in each quadrant.

16-Point Unit Circle



Using the (x, y) coordinates in the 16-point unit circle and the definitions in the Key Concept Box at the top of the page, you can find the values of the trigonometric functions for common angle measures. It is helpful to memorize these exact function values so you can quickly perform calculations involving them.